

Analysis of RR Lyrae Variable Stars

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Abstract

RR Lyrae stars have been known for being radial pulsators for quite some time. RR Lyrae are in the instability strip of the Hertzsprung-Russell Diagram (HRD) and can be classified as RRab, RRc, and RRd. Most of the analysis is done with RRc stars which are characterized as first-overtone pulsators. Interestingly, many RRc stars also exhibit a non-radial pulsation mode (denoted Fx) and it is currently not known what the mechanism is that drives this non-radial mode. Investigations into this mechanism has led to a possibility that the non-radial pulsation mode is driven by magnetic fields built up by turbulent flow in the hydrogen and helium ionization zones[1].

1 Introduction

It is known that Radial pulsation works through the κ -mechanism (where κ denotes the opacity). In this process, the partially ionized helium zone collapses under gravity below equilibrium which leads to an increase of opacity but the temperature remains relatively constant because opacity is not obeying Kramer's law:

$$\kappa \propto \frac{\rho}{T^{3.5}} \quad (1)$$

Instead the opacity is dominated by electron scattering and thus the opacity is independent of temperature. The increased opacity then traps heat from the radiative zone building pressure that eventually pushes the partially ionized helium zone outwards past equilibrium. Then the Helium layer cools allowing recombination of electrons to the Helium ions and lowers the opacity. Again the Helium zone begins to collapse under gravity falling past the equilibrium point and the cycle repeats itself.[2]

For the Fx mode, finding how it changes and whether there is any periodicity in that change can help characterize the Fx mode which can help narrow down what the driving mechanism is. I will be using Period04[3], a computer program used for statistical analysis of large astronomical time series using fourier transforms to get frequencies, amplitudes, and phases from light curves. Using various light curve observations taken by the Transiting Exoplanet Survey Satellite (TESS)[4] I can upload time strings of RRc light curves into Period04.

Using these light curves I split the data of a sector (one time string file) into four sections with 20 cycles in each group if possible. I go through a process of prewhitening which filters out the primary radial pulsation mode and begins constructing a model with that. I then pick out the Fx mode and record its amplitude and with enough data we can see how the Fx mode is changing. I also closely analyze the subharmonics of Fx which is $(1/2)Fx$ and $(3/2)Fx$ because of a possibility of a relationship between them. Other combinations of Fx also exist in the fourier spectrum generated by Period04 but they will not be considered.

2 Procedure

As previously described I am using Period04 to conduct analysis on RRc variable stars. A particular star I am interested in is XX Dor where we have 7 months of observations plus an additional 12 months of observations done on it. XX Dor features an Fx mode which commonly splits into multiplets as shown below:

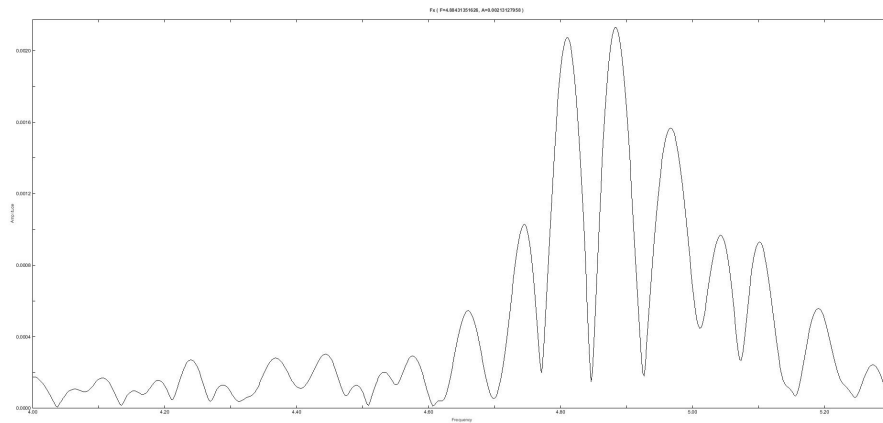


Figure 1: Fourier spectrum after prewhitening F1 (radial mode) out showing the multiple peaks corresponding to Fx

Before even using Period04 the light curves are in fluxes versus julian date but it is convention to use magnitudes to represent light intensities. There is also a known issue with Period04 if the Julian dates are not initialized to zero then the phases from Period04 will not come out correctly. This might be because Period04 generates a model of the light curve starting from zero and so it calculates phases starting from zero days. Typically, julian dates start at a 1000 days or more roughly speaking. I do all of this using a python program that accepts a data file, sets the Julian date approximately to zero by cutting off the number left of the decimal point, and doing the appropriate conversions of flux and flux errors to magnitudes and magnitude errors. Additionally, the

program also sets every sector, which is one data file containing about a month of observations, to the same zero-point. This is to ensure there is no deviation of amplitude values because of sectors not having the same relative zero-point like in this figure:

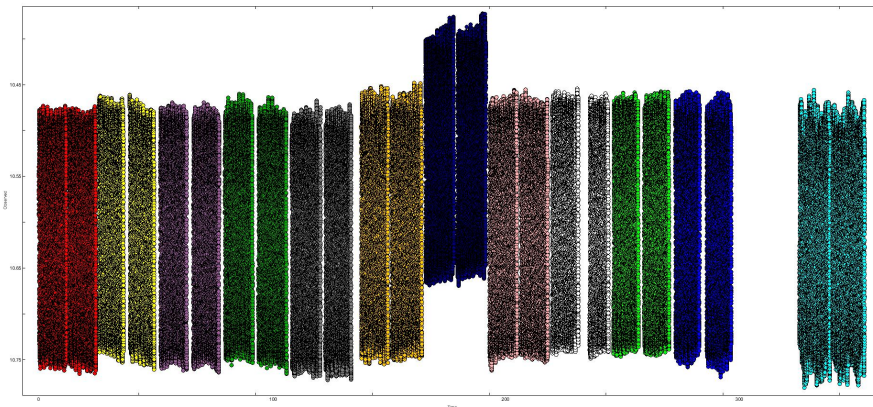


Figure 2: A real example of sectors highlighted in different colors being significantly shifted from each other (particularly the 7th sector in this case)

I then split these sectors into smaller groups but caution is necessary when doing so. If the data is split up too much and analyzed individually then the peaks generated by the fourier transform widen, taking the form of a gaussian distribution. If these peaks widen enough they can overlap with each other especially in the circumstance of multiplets with the peaks very close together. A minimum of about 18 cycles of the light curve is necessary in order to avoid this problem. Here is an example of what a split sector might look like:

On the other hand, we want to be able to split the data and measure the amplitude of F_x in each group to see how it changes with time. I then analyze each group individually beginning with the prewhitening process. First the fourier transform gives us F_1 which is then fitted by `Period04` using this formula:

$$Z + \sum A_i \sin(2\pi(\Omega_i t + \Phi_i)) \quad (2)$$

where Φ_i denotes the i th phase, Ω_i denotes the i th frequency, and Z denotes the zero point. We need F_1 and its integer multiple harmonics in order to complete prewhitening. Since there are an infinite number of harmonics we utilize a signal-to-noise ratio (SNR) built into `Period04` that tells me when the signal is no longer distinguishable from noise and thus could be ignored. Once an SNR less than 5 is achieved then enough harmonics have been calculated and the prewhitening process is done.

Once prewhitening is complete I move on to F_x . By using the fourier transform to calculate residuals we find F_x which is typically the largest peak still

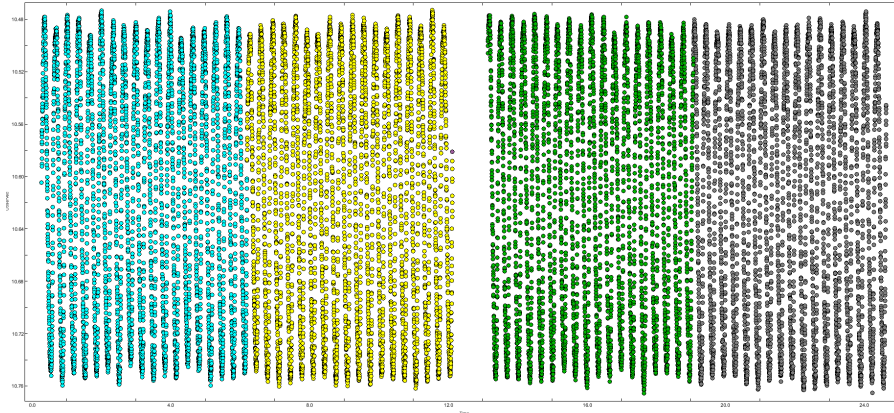


Figure 3: A sector after appropriate divisions have been made

left over, however, as we will see sometimes it is not the largest. This is due to F_x sometimes splitting into multiplets as shown in Fig.1. If the F_x mode is split into multiplets I collect all of them in Period04 and I add the amplitudes together treating F_x as a superposition of this smaller amplitudes. I also average the frequencies to compare the values with singular frequencies in the non-multiplet case. Using the same methods, I then find the $1/2F_x$ and $3/2F_x$ modes and I use the SNR to test if their amplitudes are distinguishable from noise using the same ruling as before. Once I have obtained all of the frequencies I use Period04 and run a monte carlo simulation to compute errors in the amplitudes and phases. I keep the frequencies fixed since they are not what we are primarily interested in. After I have completed this process I export the frequencies, amplitudes, and phases together, the non-initialized julian dates, and the errors in three separate files. I repeat this entire process for every grouping until everything in the data set has been analyzed and then feed the data files into a new python script. This python script generates a table of different values and does all non-Period04 calculations such as averaging the frequencies and adding the amplitudes for multiplets of F_x . It additionally averages the julian dates for each group for the purposes of plotting the amplitudes. We're also interested in the ratios A_x/A_1 , $(1/2A_x)/A_1$, and $(1/2A_x)/A_x$ (the A 's denote the amplitude) which python can also compute.

3 The Fourier Transform

As stated above Period04 utilizes a fourier transform to find the different modes associated with a light curve. In particular we use a discrete fourier transform of the form:

$$f(\omega) = \sum_{j=1}^N X(t_j)e^{-i\omega t_j} \quad (3)$$

Where X denotes the data at time t_j and ω denotes the frequency. In this discrete fourier transform the components leave only specific frequencies while the other's cancel out in the summation[2]. As the number of observations approaches infinity i.e. $N \rightarrow \infty$ the remaining frequencies become delta functions. Together these delta functions $\delta_j(\omega)$ form a function known as a dirac comb shown below:

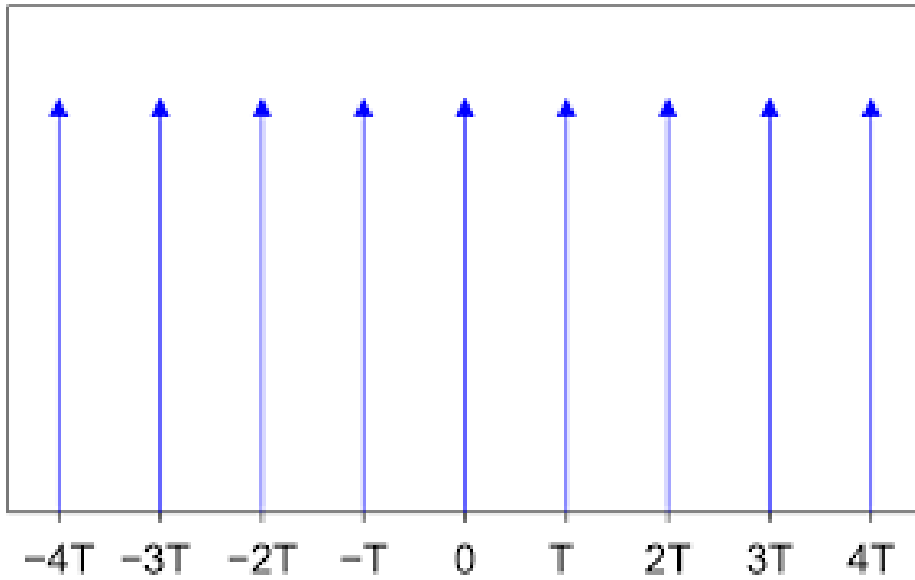


Figure 4: An example of what a Dirac Comb looks like visually. This image was acquired from Wikipedia

References

- [1] Richard B. Stothers. A new explanation of the blazkho effect in rr lyrae stars. *The Astrophysical Journal*, 2006.
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