Percolating Conductive Systems and Critical Exponents

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1 Introduction

We are modeling a conductive system as a percolation network to study critical phenomena. In our model of percolation, a network in a square lattice consists of conducting "bonds" or "links" which are joined by "sites" or "nodes". The links between each two sites may be connected with probability p, or not connected with probability 1 – p, and they are assumed to be independent of each other. The possibility that a connected path exists from top to bottom or from side to side is dependent on whether p is above or below the *percolation threshold*, the probability at which a connected path may manifest. Percolation has a singularity at the critical point $p = p_c$ and many properties behave as of a power-law as p approaches p_c. For the square lattice the percolation threshold is exactly $p_c = \frac{1}{2}$. For $p_c < \frac{1}{2}$ there is no such path, and for $p_c > \frac{1}{2}$ there is a greater than zero chance that there is a connected path.

In our conductive system the links represent conductors, with specified conductance; equal to 1(connected link) or 0(missing link). The boundaries(edges) of the square will either be grounded(V = 0) or not allow current to exit. At a chosen node inside the boundaries the voltage is specified to be V = 1. The voltages on the other sites then give rise to currents on the conductive links, and these currents have to balance at every interior site, so that just as much flows into the site as flows out(according to Kirchhoff's Laws). This defines a large set of linear equations that determines the voltages at the interior sites and the total current that flows through the network. From that point we use the set of linear equations to find the total conductance – the total current entering or leaving the network.

Now there are different questions we can pose: one can apply a voltage at one point and provide a grounded point somewhere else, and determine the current; or the grounded part could be one edge of a large square; the top edge could be at one potential and the bottom edge at another; or we could study the case with a rectangle. In each case there has to be long connected paths for there to be conductance; the magnitude of the conductance will depend on the length and whether there are many alternate routes or just a few.

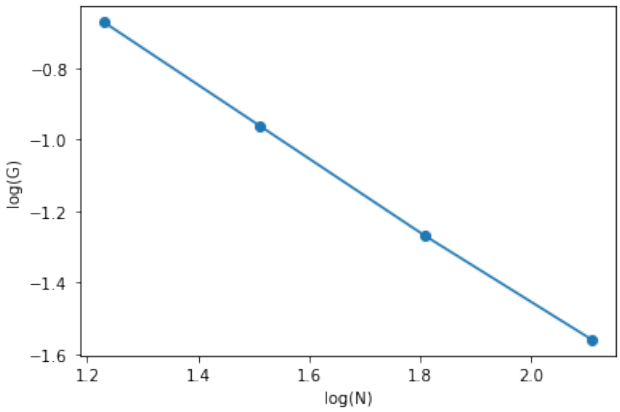
2 Procedure

The program I will be using to study this model, written by Dr. Straley, has a variable p that is the fraction of internal bonds that have unit conductance. As earlier mentioned, the percolation threshold for the square lattice is exactly $p_c = \frac{1}{2}$, so we will run our simulations with $p = \frac{1}{2}$. This will be our method of studying the percolation problem and study how physical properties behave with system size N. Our first goal is to gather data in the case where we apply a voltage to middle of the lattice and calculate the average conductance. From there we will consider other cases that were mentioned above. We plan to run many simulations and gather data using the LCC cluster. With our knowledge of the percolation threshold it is possible for us make predictions for how the conductance will behave. The conductivity exponent, t, describes how the electrical conductivity G goes to zero as the percolation threshold is approached when $p > p_c$,

$$G \sim |p - p_c|^{-t}$$

[2]

From the data we will plot $\log(N)$ versus $\log(G)$ where N is the side length of our square and G is our average conductance. The slope of the curve is the exponent, t, that describes the power law of the conductance with respect to the size of the lattice. The behavior is related to what was described above by replacing $|p - p_c|^{-t}$ by N. Thus, the conductance is given by $G \sim N^{-t}$. The critical exponent has generally been accepted to be $t \approx 0.97$ [1] but we perhaps think it could be $t = 1$.



Dr. Straley and I ran some preliminary simulations with N = 17, 32, 65, 128. The slope of the curve is approximately -1.01, which is above our predicted t, but memory overflow issues in the program kept us from simulating larger values of N. After we fix that issue we plan to simulate lattices up to potentially N $\sim$ 1000.

References

[1] CJ Lobb and DJ Frank. "Percolative conduction and the Alexander-Orbach conjecture in two dimensions". In: *Physical Review B* 30.7 (1984), p. 4090.

[2] Joseph P. Straley. "Percolation Structures and Processes". In: *Annals of the Israel Physical Society* (Vol. 5).