Phy 520: Problem Set 2
(Due: September 21, 2018)

1). For surface tension waves in shallow water, the relationship between frequency ν and wavelength λ is given by

$$\nu = \sqrt{\frac{2\pi T}{\rho \lambda^3}} \tag{1}$$

where T is the tension and ρ is the density. Compute the magnitude of the group velocity. What is the magnitude of the phase velocity, which is given by $v_p \equiv \lambda\nu$, and how does it compare to the group velocity you have computed?

2). A relativistic, free particle of rest mass m and velocity $\mathbf{v}$ has a total energy E and momentum $\mathbf{p}$ given by

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \quad ; \quad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} \,. \tag{2}$$

a) Show that these expressions satisfy $E = \sqrt{m^2c^4 + |\mathbf{p}|^2c^2}$.

b) Determine the relationship between the velocity $\mathbf{v}$ of the particle and the group velocity $\mathbf{v}_g$ of its associated de Broglie wave.

c) Determine the magnitude of the phase velocity $\mathbf{v}_p$ of the particle. Show that it exceeds the speed of light c but that $v_g v_p = c^2$. Explain why this is not a challenge to special relativity.

3). Griffiths and Schroeter, Problem 2.20.

4). Griffiths and Schroeter, Problem 2.21. f) Without doing an explicit calculation, what is $\langle x^{17} \rangle$? Explain your reasoning.

5). Use the uncertainty relation to *estimate* the ground-state energy of a one-dimensional harmonic oscillator. The energy is given by

$$E = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \,. \tag{3}$$

Hint: The most likely value of x^2 (or of p^2) occurs when E has an extremum.

6). Nuclei of a size of roughly 10 fm frequently emit electrons with energies in the range of 1-10 MeV. It was once thought that electrons "lived" inside the nuclei. Use the uncertainty relation to show that electrons of such energies could not be contained inside the nucleus.