Phy 520: Problem Set 4
(Due: October 12, 2018)

1). (*20 points*) Consider a potential consisting of a one-dimensional box with infinite walls located at $x = -a/2$ and $x = a/2$.

a) Noting the shapes of the eigenfunctions for this problem, deduce the energy eigenvalues of a new box potential in which the right-hand wall is located at $x = 0$. You ought not solve the Schroedinger equation to come to an answer.

b) Suppose a particle in the original box happens to be localized in the left-hand side of the box, so that $\psi(x)$ is non-vanishing and has the value

$$\psi(x) = \sqrt{\frac{2}{a}} \quad \text{if } -a/2 < x < 0. \tag{1}$$

Do you think the particle will remain localized at later times? Explain why or why not. What is the probability that an energy measurement yields the energy eigenvale associated with an particular value of n?

c) Noting that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \tag{2}$$

show that the probability you calculated in part b) for fixed n, $\mathcal{P}_n$, satisfies

$$\sum_{n=1}^{\infty} \mathcal{P}_n = 1. \tag{3}$$

2). Return to the box potential first described in Problem # 1, but now suppose the particle has the initial wave function

$$\psi(x, 0) = A \left(\sin \left(\frac{\pi x}{a} \right) \right)^7. \tag{4}$$

a) Calculate the normalization A.

b) What is the form of $\psi(x, t)$?

c) What is the probability that an energy measurement yields E_2? What is the probability of measuring E_3? Can you explain, by appealing to physical principles, the difference in your results?

Hint: You may find it helpful to rewrite $\sin(x)$ in terms of complex exponentials.

3). What is the flux associated with the particle described by the wave function

$$\psi(x) = u(x)e^{ikx} , \tag{5}$$

where $u(x)$ is a real function and k is a real number? How does your result change if k is a complex number?

4). The time-independent wave function of a system, written in position space as $\psi(x)$ and in momentum space as $\psi(p)$, satifies

$$\psi(x) = \int_{-\infty}^{\infty} dp\, \phi(p) \frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}} \; ; \; \psi(p) = \int_{-\infty}^{\infty} dx\, \psi(x) \frac{e^{-ipx/\hbar}}{\sqrt{2\pi\hbar}} \tag{6}$$

a) Construct the eigenfunctions of $\hat{p}$, $u_p(x)$, as well as the eigenfunctions of $\hat{x}$, $u_x(p)$. Are the eigenvalues in each case continuous or discrete?

b) Use the results of part a) to discuss the physical interpretation of Eq. 6) in terms of the expansion postulate.

c) If $\psi(x) = (\alpha/\pi)^{1/4} \exp(-\alpha x^2/2)$, then what is the probability that the momentum is in the range of $(p, p + dp)$?