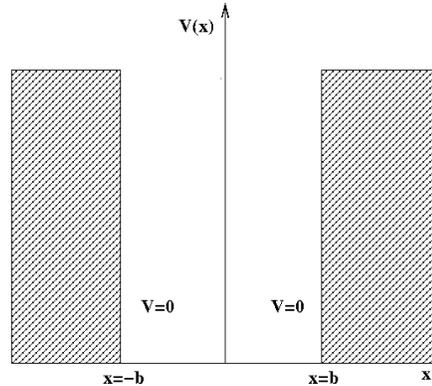


Phy 520: Problem Set 5
(Due: October 31, 2018)

1). (20 points) In this problem we consider an infinite, square-well potential in one spatial dimension, such that $V = 0$ if $|x| \leq b$ and V is **infinite** everywhere else, as illustrated in the figure:



- a) Evaluate Δx and Δp for each eigenstate n in this potential. (Recall that $(\Delta x)^2 \equiv \langle x^2 \rangle - \langle x \rangle^2$, with an analogous expression for Δp .)
- b) How do Δx , Δp , and $\Delta x \Delta p$ change with n ? Considering the shapes of the eigenfunctions with n can you explain the results you have found?
- c) Now suppose the wave function of a particle in the potential is given by $\psi(x) = A(\sin(\pi x/b))^3$ for $|x| \leq b$ (and zero otherwise), where A is a normalization constant. What are the possible results of an energy measurement? Determine $\langle x^2(t) \rangle$ before and after an energy measurement.
- d) Suppose you measure the energy at t_1 for the system described in part c). If you were to measure the energy again at a later time t_2 , what would be the possible results of an energy measurement? If the result of the first energy measurement were known, how does that impact the possible result of a second energy measurement?
- e) What is the value of $\langle [\hat{x}, \hat{p}^{14}] \rangle$ for the system described in part c)? Explain your answer.

2). Consider our graphical analysis of the bound states in a finite square well of depth V_0 and width a . Determine:

- a) The condition on V_0 and a that there is at most one bound state in the problem.
- b) The condition on V_0 and a that there is at most four bound states in the problem.
- c) Suppose the potential parameters are such that the third bound state is just barely bound. What can you say about the the binding energy of the first and second bound states?

3). In this problem you are asked to set up the solutions of the Schroedinger equation in each region so that only the matching conditions at the interior boundaries is left to be done. You are not asked to do this — just to set the solution up so that it could be done.

a) Consider the potential in which an infinite wall exists at $x = 0$ and a finite potential barrier of height V_0 extends from $x = a$ to $x = b$ with both a and b greater than zero. The potential is zero for all other positive values of x . Set up the solution under the condition that a flux of magnitude $\hbar k/m$ is incident from the right (for $x \gg b$) with the energy $E = (\hbar k)^2/2m$ such that $E < V_0$.

b) Consider a double finite potential well of depth V_0 . One well extends from $-a$ to $-b$, and the other well extends from b to a . Note that b and a are both positive with $b < a$. What is the form of the solution for $E < 0$?

4). Griffiths and Schroeter (!), Problem 2.34.

5). Griffiths and Schroeter (!), Problem 2.43.

6). Griffiths and Schroeter (!), Problem 2.53.

7). Consider the potential

$$V(x) = -\eta(\delta(x - a) + \delta(x + a)), \quad (1)$$

where η and a are positive constants.

- a) Determine the solution to the time-dependent Schroedinger equation for an incident plane wave from the left with momentum $\hbar k$ and flux $\hbar k/m$, where m is the mass of the particle.
- b) Compute the transmitted flux for the solution of part a).
- c) How are the coefficients of the reflected and transmitted waves in part a) related to the S-matrix elements for this problem? Can bound states exist for this potential? How many are there? Explain how you can tell.