Phy 632: Problem Set 2

(Due: February 17, 2011)

8). a) Consider a box of 1 Ω resistors. Each of these resistors actually has a probability density function for its resistance of $P(r) = \sqrt{\alpha/\pi}\exp(-\alpha(r-1)^2)$. These probabilities are all statistically independent. Suppose N of these resistors are connected in series from point a to b. What is the probability density function for the combined resistance from a to b? (*Hint:* You should assume that $\alpha \gg 1$ throughout.)

b) Calculate this distribution's characteristic function and lowest three cumulants.

c) Determine the probability density function in the $N \to \infty$ limit. Does your result agree with the central limit theorem?

d) If you connected the resistors in parallel would your answers to part c) change? Why or why not?

9). A gas contains equal numbers of A particles and B particles. The A particles and B particles, respectively, have energy distributions $P_A(E)$ and $P_B(E)$, where $0 \leq E < \infty$. Given that a particle has an energy larger than E_0, write an expression for the probability that it is an A particle.

10). A flea jumps randomly along a straight line in steps of length d, positive or negative, starting from the origin. Its probability of making a positive jump is p, that of a negative jump $1-p$.

a) At time $t_n = n$ (so that n is also the total number of steps taken) the flea is at a location $x_k = kd$ with probability $P(n,k)$. Compute $P(n,k)$.

b) Calculate the expectation value of the displacement from the origin and show that the mean square deviation from this value goes as n.

c) Show that, as n and k approach infinity, that your results for parts a) and b) agree with the predictions of the central limit theorem.

11). Kardar, Ch. 2, Problem #8.

12). Kardar, Ch. 2, Problem #9.

13). Determine the set of P_i^* which maximize the entropy S, where

$$S = -k_B \sum_j P_j \ln P_j \tag{1}$$

subject to the constraint that $\sum_j P_j = 1$. Do the P_i^* depend on i? Repeat the maximization procedure now with both $\sum_j P_j = 1$ and $\sum_j E_j P_j = E$, where E is fixed, as constraints. What is P_i^* now?

14). Consider the complementary error function erfc(x) defined by

$$\begin{aligned} \text{erfc(x)} &\equiv \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2) dt \\ &= \frac{2}{\sqrt{\pi}} \exp(-x^2) \int_0^\infty \exp(-2tx) \exp(-t^2) dt \end{aligned} \tag{2}$$

Although the integrand attains its maximum value at an end point, we can still use the ideas of lecture to obtain an asymptotic expansion for large values of x. To proceed, expand $\exp(-t^2)$ in a power series about $t = 0$ and integrate term by term to obtain an asymptotic series for large x. Denoting the partial sum of this series through term n as S_n, compare the exact value of erfc(x) with S_2 and S_3 for $x = 2$ and $x = 4$ — report the comparison as a percentage difference from the exact value.

Note erfc(2) is 0.004677735... and erfc(4) is 0.0000000154173....

15). Consider the classical Hamiltonian $H = p^2/2m + m\omega^2 q^2/2$, the harmonic oscillator in one spatial dimension.

a) Determine the values of p and q at time t, given their values p_0 and q_0 at time $t = 0$.

b) Show explicitly that the Jacobian $J(t, t')$, as introduced in lecture, is unity for all t, t' and thus that the phase volume is conserved (Liouville theorem).