Phy 632: Problem Set 6
(Due: April 12, 2011)

31). Suppose two ideal gases, each with the same N, V, and T, are brought into thermal contact. If the two ideal gases are not allow to mix, what is the entropy of the two-gas system? Now suppose the gases are allowed to mix. Compute the entropy (i) if two gases are comprised of the same monoatomic gas and (ii) if the two gases are comprised of two different monoatomic gases.

32). Compute the number of quantum states per unit energy, $\rho(\varepsilon)$, as a function of energy for a particle in a box in d-dimensional space. Note that the solid angle in d-dimensional momentum space is $S_d = 2\pi^{d/2}/\Gamma(d/2)$. For what value of d is $\rho(\varepsilon)$ energy independent?

33). Show that $PV = 2E/3$ for a nonrelativistic perfect gas. What is PV/E if the particles are extremely relativistic? Are your results valid beyond the low density, high temperature limit? Explain.

34). The sign of the Joule-Thompson coefficient determines whether a gas can be cooled, and ultimately liquified, upon its passage through a porous plug. a) Compute the Joule-Thompson coefficient $\partial T/\partial P|_H$, where H is the enthalpy, for a classical, perfect gas. b) Find the leading-order quantum corrections for Bose-Einstein and Fermi-Dirac statistics. Can both types of perfect gases be condensed?

35). A two-dimensional atomic gas is adsorbed onto a plane surface with binding energy $-\varepsilon_0$; the adsorbed atoms move freely in the two-dimensional plane. The gas is in equilibrium with its own three-dimensional vapor which has pressure P and temperature T. Determine the number adsorbed per unit area as a function of P and T in the classical limit. Also find the leading corrections for quantum degeneracy in the adsorbed part. Under what circumstances might the three-dimensional gas become degenerate?

36). Consider a system of molecular H_2 in the canonical ensemble. Compute and sketch the rotational/nuclear contribution to the heat capacity at fixed volume at low temperatures (that is, for T no greater than 100°K) for (a) an equilibrium mixture of ortho- and para-H_2 and (b) for a mixture in which the ratio of ortho- and para-H_2 is fixed to its value in the high temperature limit ($\beta\overline{B} \ll 1$). In the absence of a catalyst, the conversion of ortho- to para-H_2 at moderate temperatures is sufficiently slow that the second calculation is germane to a comparision with empirical data. (Practically one starts with a system at room temperature and then cools it down.) What would the rotational/nuclear partition function be for the case of molecular D_2? The deuteron is a spin-one nucleus, composed of one neutron and one proton. Note that $\overline{B}/k_B$ is 85.3°K for H_2 and is 42.7°K for D_2.

Hint: For the low temperatures of interest, the partition functions are well-approximated by the first few terms in the sums....