

## Class 37: Charging and Discharging RL Circuits

# Course Evaluation:

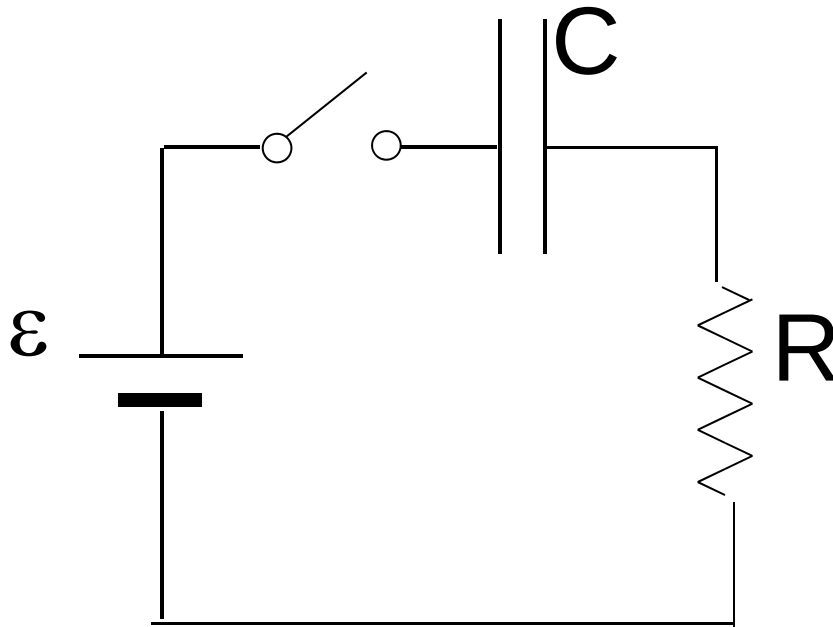
1. Starts Wednesday, ends Dec 10<sup>th</sup>.
2. Go to <http://pa.as.uky.edu/>
3. Click at “UNDERGRADUATES” in the top menu and then choose the first item: Physics & Astronomy Course Evaluations
4. Follow instructions from there.
5. Make sure remember or write down any given key or password. You need this to re-enter the system if you cannot finish the evaluation in one time.

From Class 22

Slide #3

# RC Circuits – Charging

← Charge



$$\varepsilon = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R \frac{dq}{dt} = \varepsilon$$

$$\Rightarrow CR dq = (C\varepsilon - q) dt$$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{CR} dt \quad \text{Integration constant}$$

$$\Rightarrow \ln(q - C\varepsilon) = -\frac{t}{CR} + K'$$

$$\Rightarrow q - C\varepsilon = Ke^{-\frac{t}{CR}} \quad (K = e^{K'})$$

$$\Rightarrow q = C\varepsilon + Ke^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = 0 \Rightarrow 0 = C\varepsilon + K \Rightarrow K = -C\varepsilon$$

$$\therefore \underline{\underline{q = C\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$$I = \frac{dq}{dt} = \frac{C\varepsilon}{CR} e^{-\frac{t}{CR}} = \underline{\underline{\frac{\varepsilon}{R} e^{-\frac{t}{CR}}}}$$

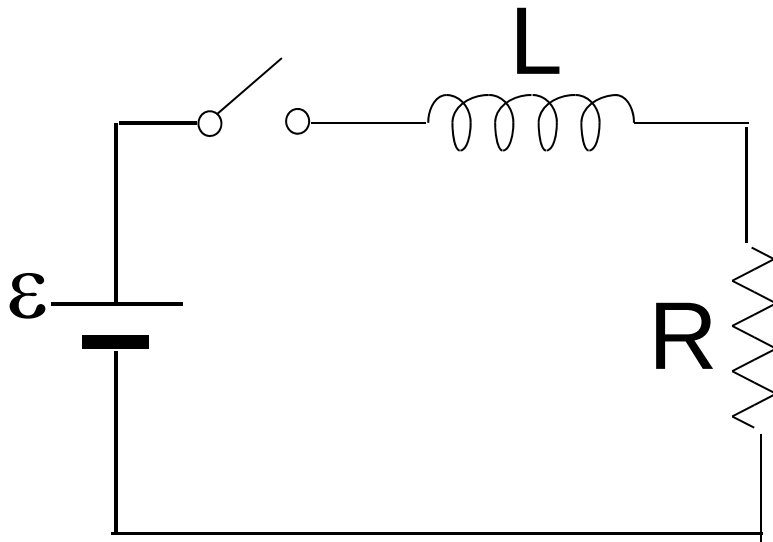
$$\Delta V_R = IR = \underline{\underline{\varepsilon e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$$\left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\} \Delta V_R + \Delta V_C = \varepsilon$$

# RL Circuits – Charging

← Current



$$\varepsilon = L \frac{dI}{dt} + IR \Rightarrow L dI + I R dt = \varepsilon dt$$

$$\Rightarrow L dI = (\varepsilon - IR)dt$$

$$\Rightarrow \frac{L dI}{\varepsilon - IR} = dt$$

Integration constant

$$\Rightarrow \frac{L}{R} \ln(\varepsilon - IR) = t + K'$$

$$\Rightarrow \ln(\varepsilon - IR) = \frac{R}{L} t + \frac{R}{L} K'$$

$$\Rightarrow \varepsilon - IR = K e^{-\frac{R}{L} t} \quad (K = e^{\frac{R}{L} K'})$$

$$\Rightarrow IR = \varepsilon - K e^{-\frac{R}{L} t}$$

$$\text{At } t = 0, I = 0 \Rightarrow 0 = \varepsilon - K \Rightarrow K = \varepsilon$$

$$\therefore IR = \varepsilon(1 - e^{-\frac{R}{L} t}) \Rightarrow \underline{\underline{I = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L} t})}}$$

$$\Delta V_R = IR = \underline{\underline{\varepsilon(1 - e^{-\frac{R}{L} t})}}$$

$$\Delta V_L = L \frac{dI}{dt} = \underline{\underline{\varepsilon e^{-\frac{R}{L} t}}}$$

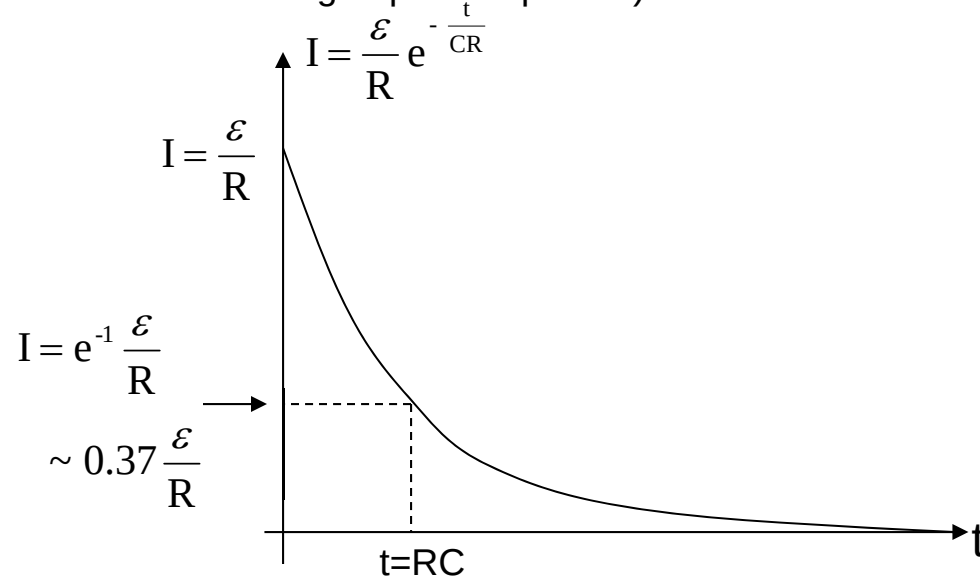
$$\left\{ \begin{array}{l} \leftarrow \\ \rightarrow \end{array} \right. \Delta V_R + \Delta V_L = \varepsilon$$

From Class 23

Slide #4

# RC time constant

$\tau = RC$  is known as the RC time constant. It indicates the response time (how fast you can charge up the capacitor) of the RC circuit.

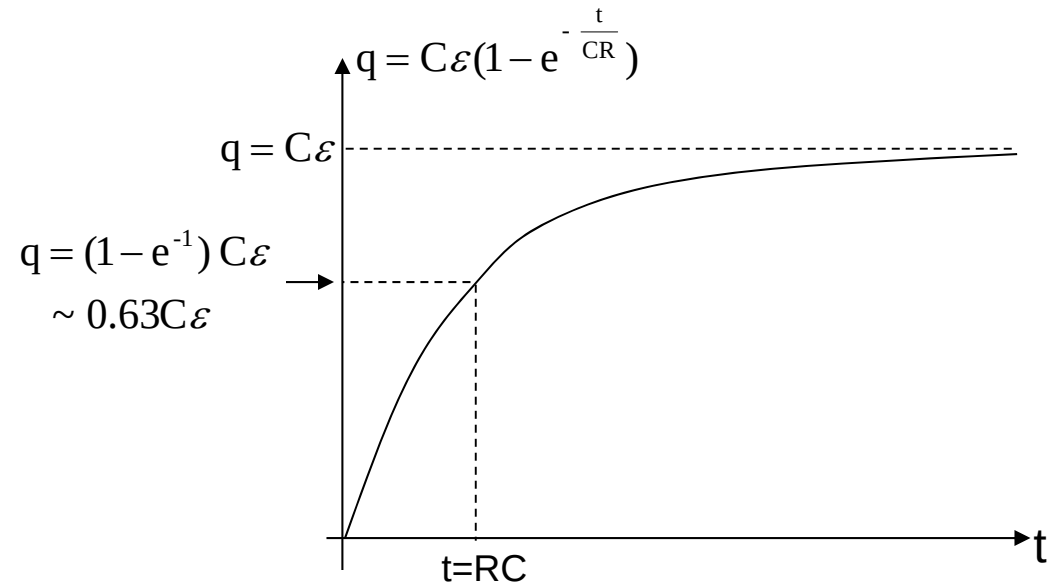


$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$

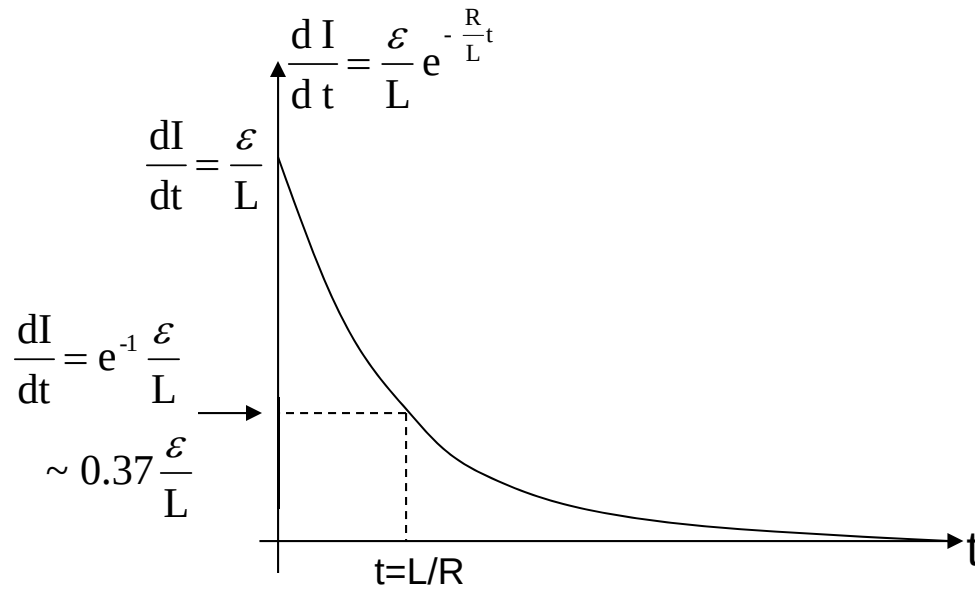
$$\sqrt{2} \approx 1.414$$
$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RC circuits



# L/R time constant

$\tau = L/R$  is known as the time constant. It indicates the response time (how fast you can up a current) of the RC circuit.



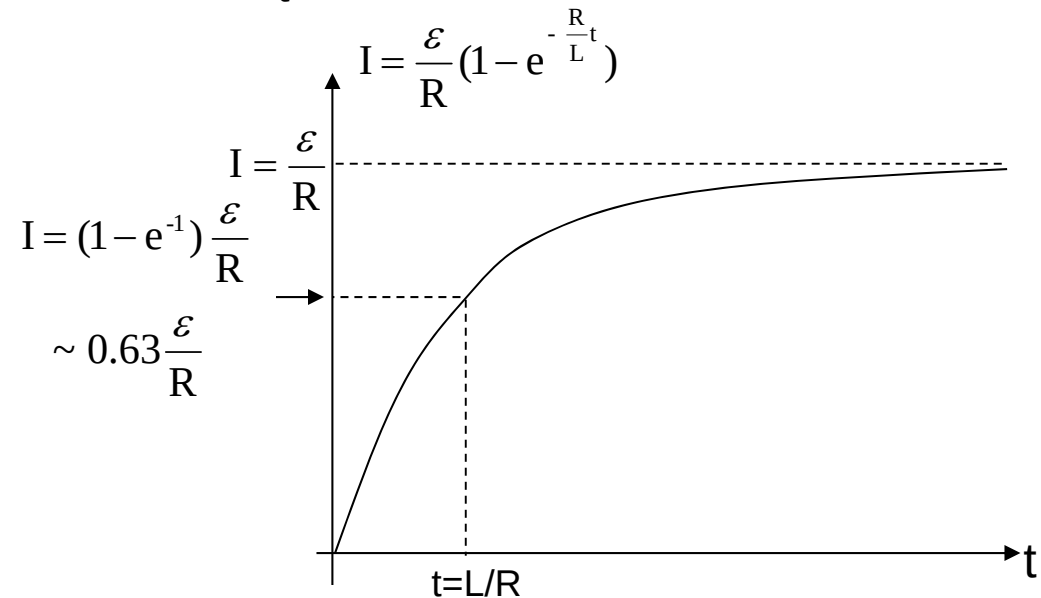
$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$

$$\sqrt{2} \approx 1.414$$

$$\frac{1}{\sqrt{2}} \approx 0.707$$

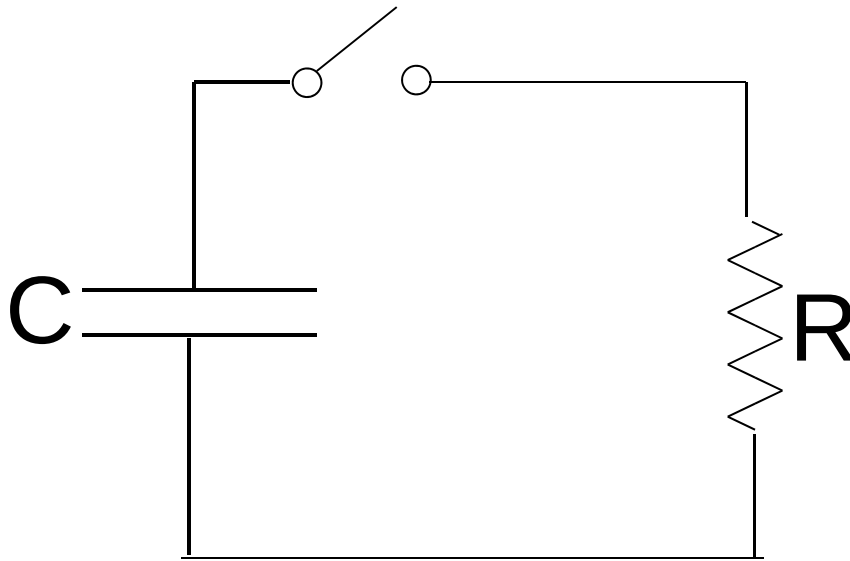
Nothing to do with RL circuits



From Class 23

Slide #5

# RC Circuits – Discharging ← Charge



$$0 = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R \frac{dq}{dt} = 0$$

$$\Rightarrow CR \, dq = -q \, dt$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{CR} dt$$

Integration constant

$$\Rightarrow \ln q = -\frac{t}{CR} + K'$$

$$\Rightarrow q = K e^{-\frac{t}{CR}} \quad (K = e^{K'})$$

$$\Rightarrow q = K e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = Q \Rightarrow Q = K$$

$$\therefore q = \underline{\underline{Q e^{-\frac{t}{CR}}}}$$

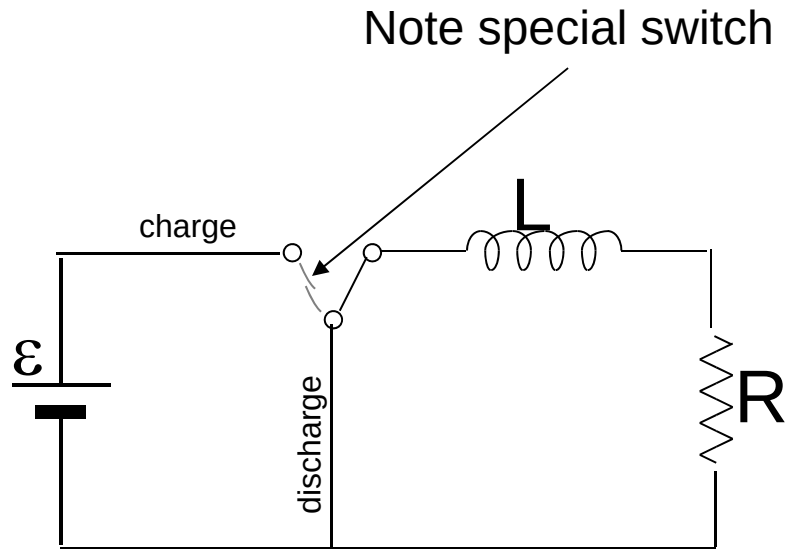
$$I = \frac{dq}{dt} = -\underline{\underline{\frac{Q}{RC} e^{-\frac{t}{CR}}}}$$

$$\Delta V_R = IR = -\underline{\underline{\frac{Q}{C} e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\frac{Q}{C} e^{-\frac{t}{CR}}}}$$

$$\left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\} \Delta V_R + \Delta V_C = 0$$

# RL Circuits – Discharging ← Current



$$0 = L \frac{dI}{dt} + IR \Rightarrow L dI = -IR dt$$

$$\Rightarrow \frac{dI}{I} = -\frac{R}{L} dt$$

Integration constant

$$\Rightarrow \ln I = -\frac{R}{L} t + K'$$

$$\Rightarrow I = K e^{-\frac{R}{L} t} \quad (K = e^{K'})$$

$$\text{At } t = 0, I = I_0 = \frac{\varepsilon}{R} \Rightarrow K = I_0 = \frac{\varepsilon}{R}$$

$$\therefore I = \underline{\underline{I_0 e^{-\frac{R}{L} t} \quad \text{or} \quad \frac{\varepsilon}{R} e^{-\frac{R}{L} t}}}$$

$$I = I_0 e^{-\frac{t}{CR}}$$

$$\Delta V_R = IR = \underline{\underline{I_0 R e^{-\frac{R}{L} t}}}$$

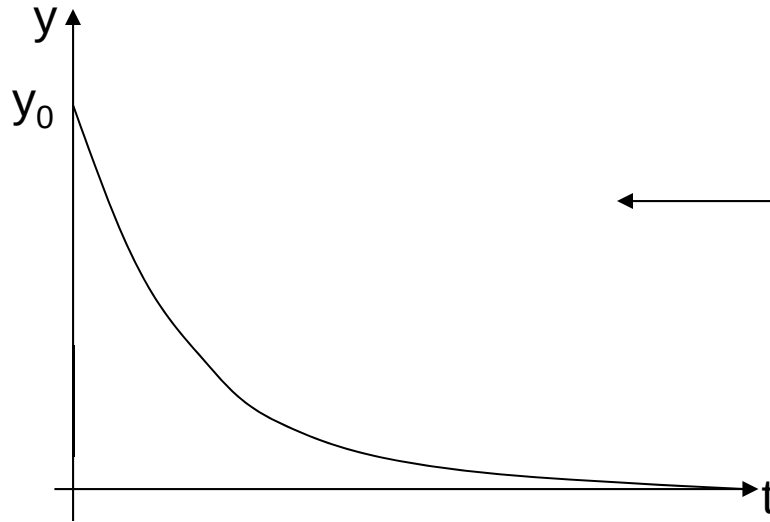
$$\Delta V_L = L \frac{dI}{dt} = \underline{\underline{-I_0 R e^{-\frac{R}{L} t}}}$$

$$\left. \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right\} \Delta V_R + \Delta V_L = 0$$



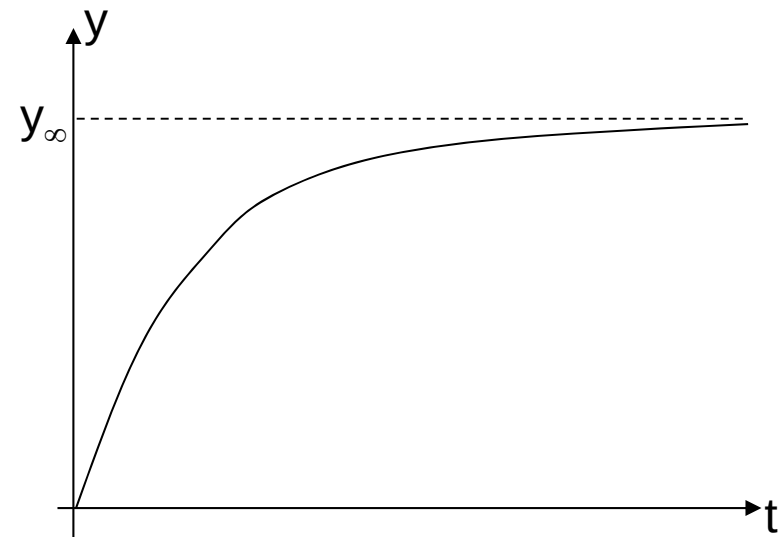
# In Summary

For both charge and discharge,  $Q$ ,  $I$ ,  $\Delta V_C$ , and  $\Delta V_R$  must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

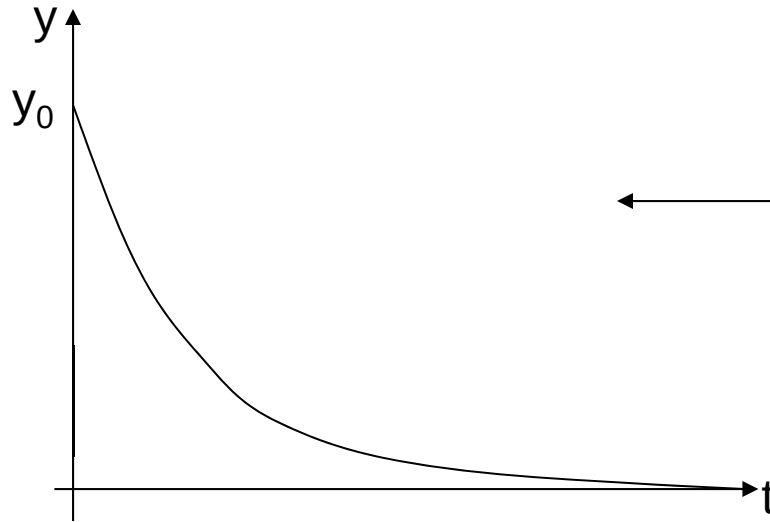
$$y = y_{\infty} (1 - e^{-\frac{t}{RC}})$$



$y$  can be  $Q$ ,  $I$ ,  $\Delta V_C$ , or  $\Delta V_R$

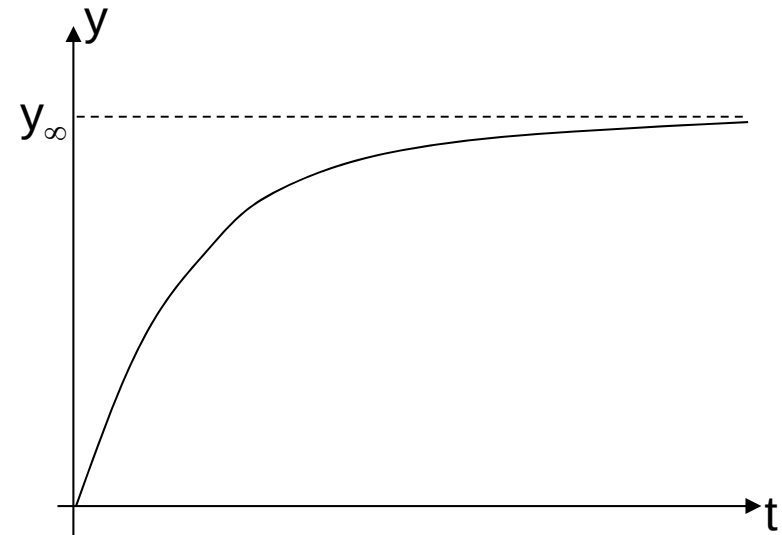
# In Summary

For both charge and discharge,  $I$ ,  $di/dt$ ,  $\Delta V_L$ , and  $\Delta V_R$  must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

$$y = y_{\infty} (1 - e^{-\frac{t}{RC}})$$



$y$  can be  $I$ ,  $di/dt$ ,  $\Delta V_L$ , or  $\Delta V_R$