Electric Potential Energy and Electric Potential

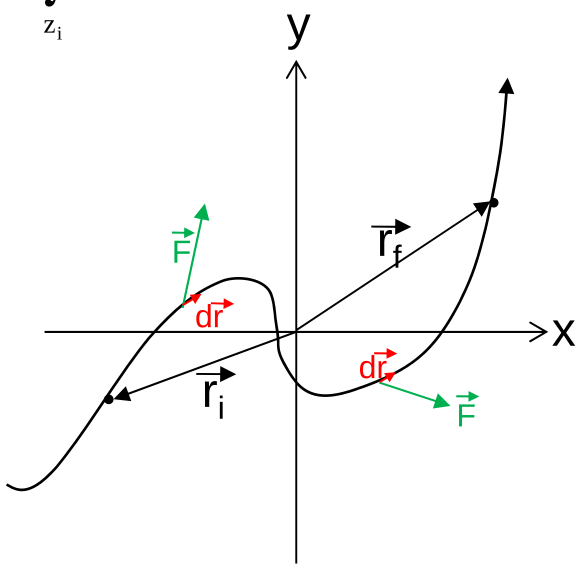
Work

Work done W by a force $\vec{F} = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$

$$\vec{F} = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

$$= \int_{x_i}^{x_f} F_x\,dr_x + \int_{y_i}^{y_f} F_y\,dr_y + \int_{z_i}^{z_f} F_z\,dr_z$$

Work done W by a force $\vec{F} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$



Potential energy

$$\Delta K = K_f - K_i \quad = \quad \text{Work done by electric force } \vec{F}_E$$

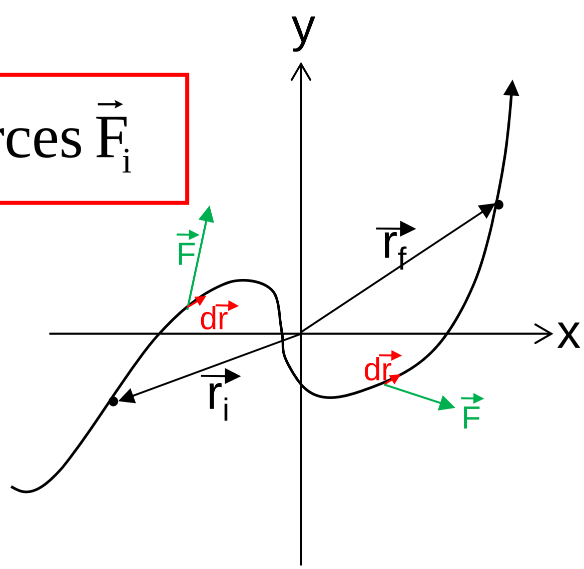
$$+ \text{ Work done by other forces } \vec{F}_i$$

Static electric is conservative, so we can define electric potential energy U_E as:

$$\Delta U_E = -\text{Work done by electric force } \vec{F}_E$$

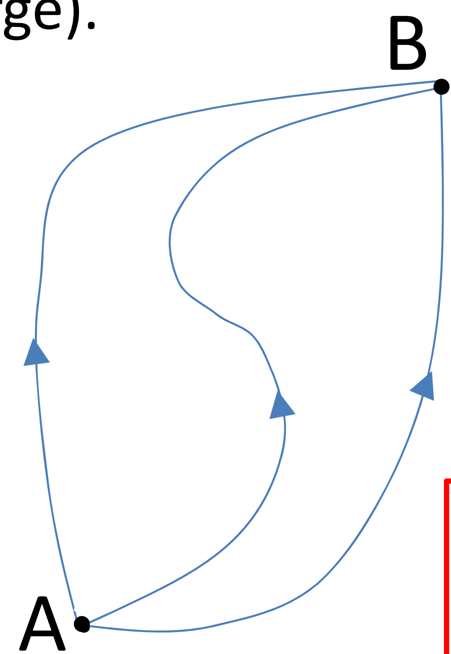
$$\therefore \quad \Delta K \quad = \quad -\Delta U_E + \text{ Work done by other forces } \vec{F}_i$$

$$\Rightarrow \boxed{\Delta K \; + \; \Delta U \; = \; \text{Work done by other forces } \vec{F}_i}$$



Electric Potential

Static $\vec{E}(\vec{r})$ is conservative, the potential difference ΔV is defined as the *negative* work done by the force $\vec{F}(\vec{r})$ (which is path independent), divided by the charge (of the test charge).



$$\Delta U = -\int_i^f \vec{F}(\vec{r}) \cdot d\vec{r}$$

Pay attention to the negative sign

$$\Delta V = \frac{\Delta U}{q} = -\int_i^f \vec{E}(\vec{r}) \cdot d\vec{r}$$

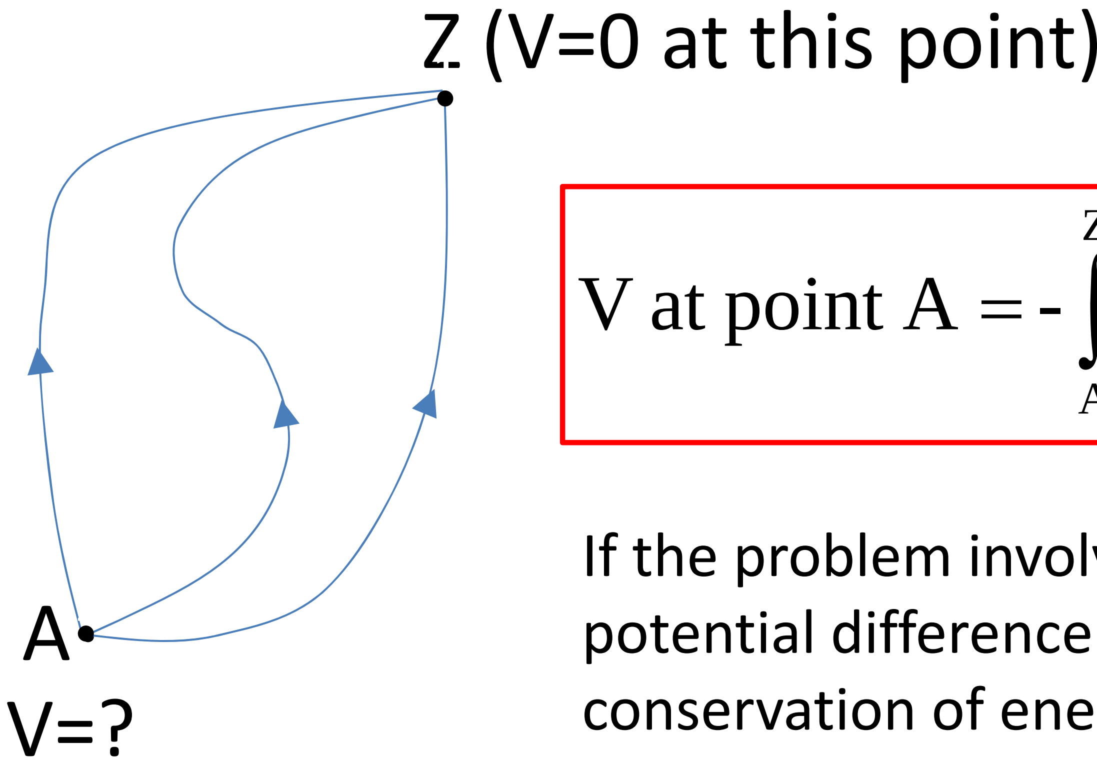
Unit of electric potential = J/C =V

Warning

In the discussion here we will assume electric (force) field is a conservative (force) field. This will not be the case if there is a changing magnetic field. We will come to this point later in the semester.

Potential Difference and Potential

If we can define a point Z in space as a point with zero potential, then the potential of all other points in space is defined.



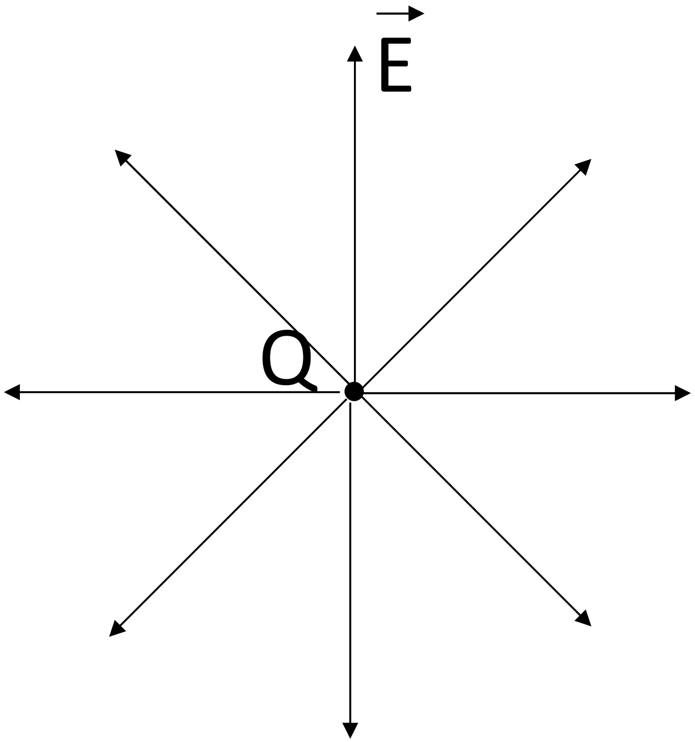
$$V \text{ at point } A = -\int_A^Z \vec{E}(\vec{r}) \cdot d\vec{r}$$

If the problem involves only potential difference (e.g. conservation of energy), the choice of this zero point is not important.

Class 13. Calculation of Electric Potential

Electric potential of a Point Charge

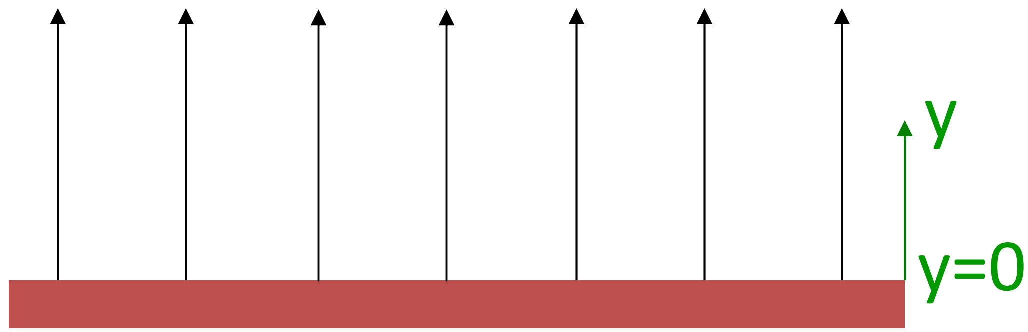
This is important because from this we can calculate the potential of any source charge assembly.



$$V = \frac{Q}{4\pi\varepsilon_0 \ r}$$

V=0 at r=∞

Electric potential of a Constant Field



$$\boxed{V = -Ey} = -\frac{\sigma}{2\varepsilon_0}y$$

V=0 at the sheet of source charges (y=0)

General Observations

1. Electric field tends to point from a high potential point to a low potential point.

2. If you release a test charge particle from rest and let it go along the field line for a short time, the particle will go from a high potential point to a low potential point if it is positive in charge. In reverse, it will go from a low potential point to a high potential point if it is negative in charge.

Calculating Electric Field from Electric Potential

Given an electric field, we can calculate the corresponding potential

$$\text{V at point A} = -\int_{A}^{Z} \vec{E}(\vec{r}) \cdot d\vec{r}$$

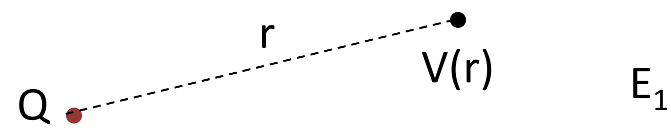
In reverse, given an electric potential, we can calculate the corresponding field:

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

Calculate the electric potential due to the source charges

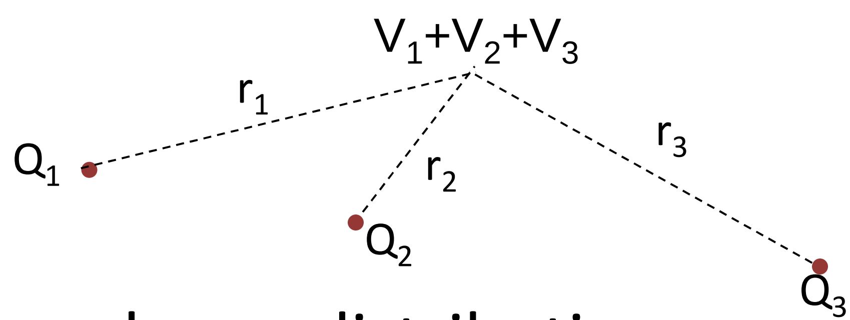
Electric potential due to a point charge:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



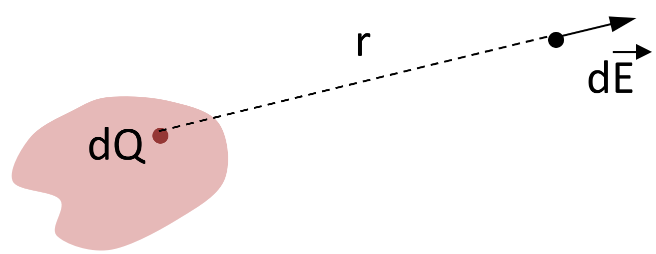
Electric potential due to several point charges:

$$\vec{E} = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{Q_i}{r_i}$$



Electric field due to continuous charge distribution:

$$V = \int dV = \int \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r}$$



Note that electric potential is a scalar, it is easier to calculate than electric field (vector).

Calculate Change in Potential Energy

If you move a small test charge (so small that it will not affect the charge distribution of the source) of charge q from point i to point f, the change in its potential energy is

$$\Delta U = q\,(V_f - V_i)$$

and now you can use conservation of energy to solve problem:

$$\Delta K + \Delta U = 0$$