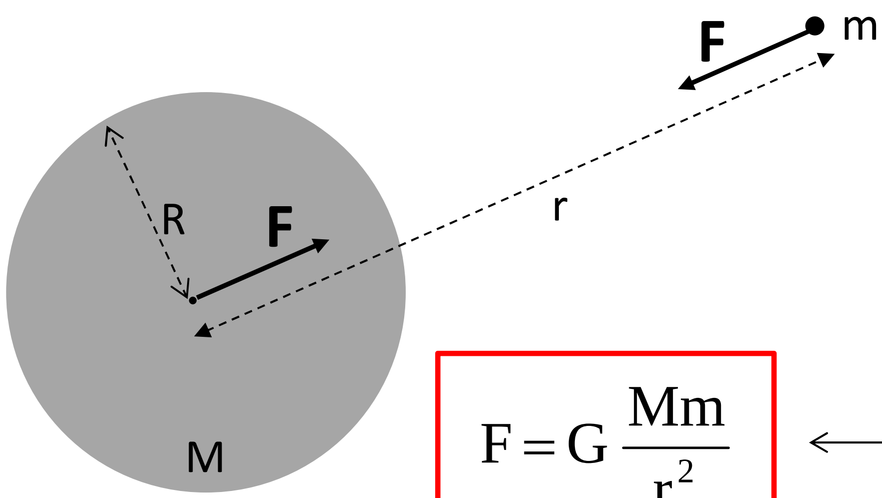
Circular motion and Gravitational Law

Gravitational attraction between a point particle and a sphere (outside)

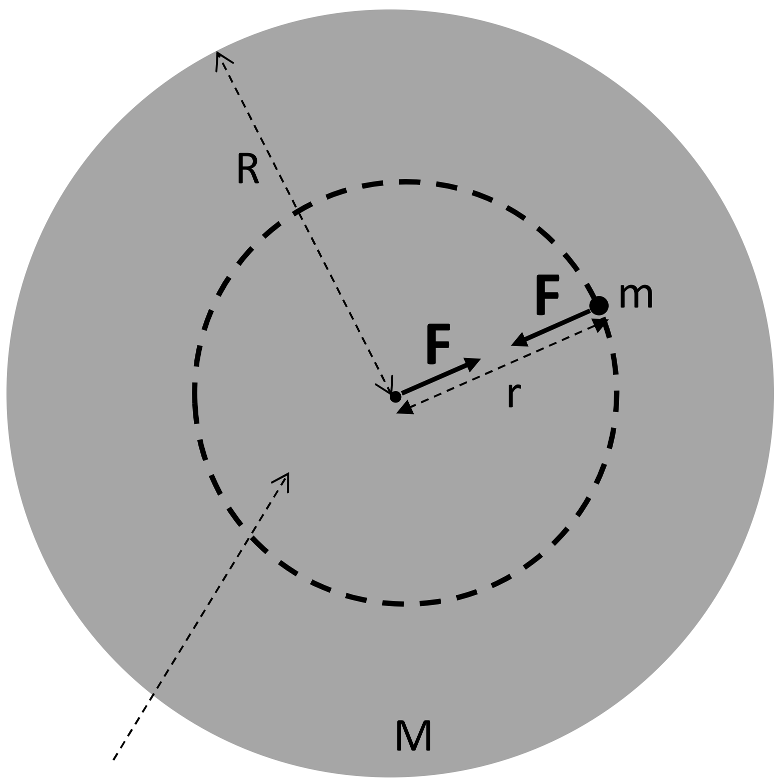


$$F = G\frac{Mm}{r^2}$$

← Magnitude equation

Always attractive!

Gravitational attraction between a point particle and a sphere (inside)



Only masses inside dotted circle (M') count!

$$F = G\frac{M'm}{r^2}$$

← Magnitude equation

Assuming uniform density

$$M' = \frac{M}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi r^3 = M\left(\frac{r}{R}\right)^3$$

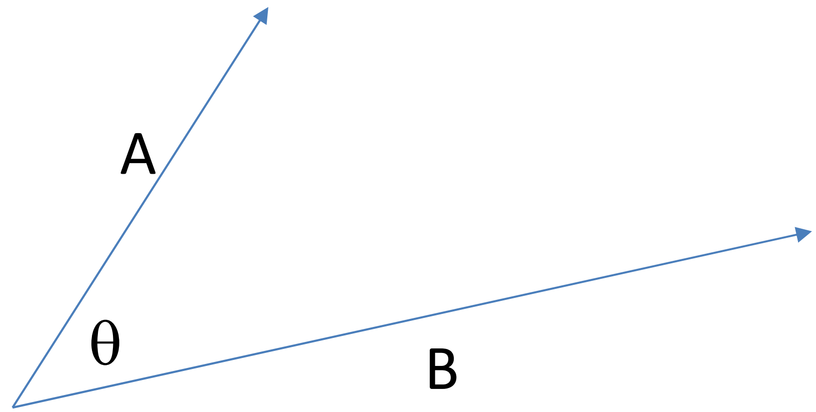
$$\therefore F = G\frac{Mm}{r^2}\cdot\left(\frac{r}{R}\right)^3 = G\frac{Mm}{R^3}\cdot r$$

Class 2: Energy and Conservative Law

Kinetic Energy

$$T = \frac{1}{2}mv^2$$

Dot product between two vectors



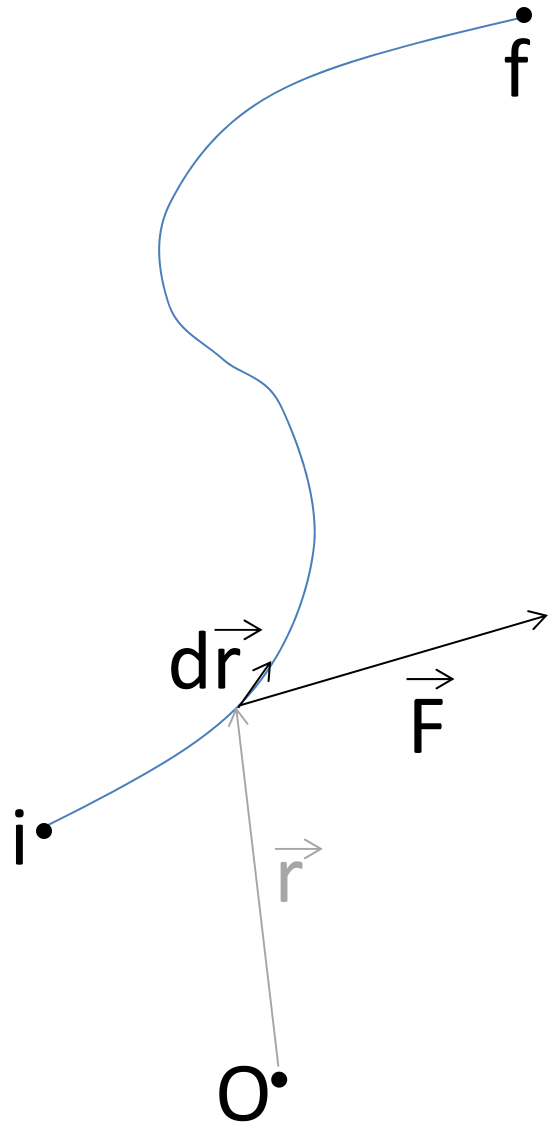
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}| \cos\theta$$

$$\sin 0^o = 0 \qquad \sin 90^o = 1$$

$$\cos 0^o = 1 \qquad \cos 90^o = 0$$

← You should know this.

Work done



Work done *by* F

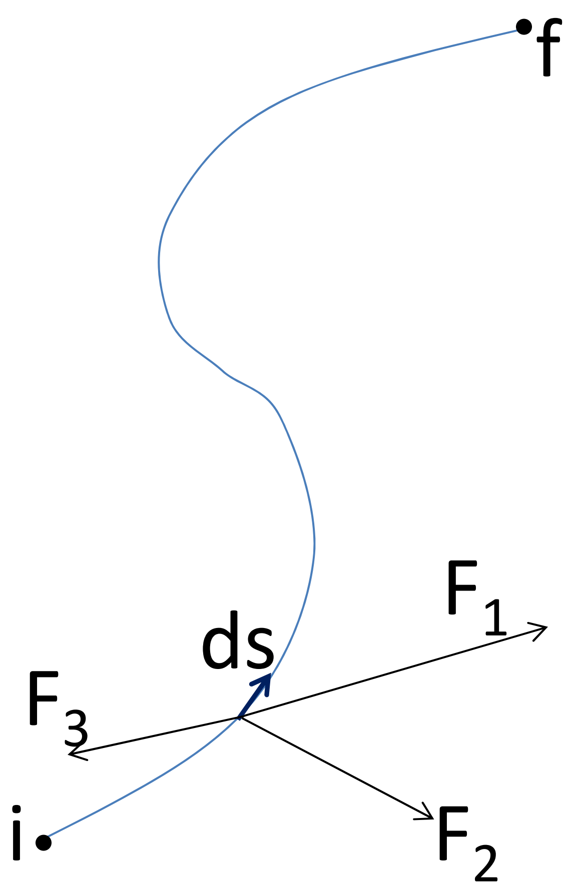
$$= W_{i\to f} = \int_i^f \vec{F}\cdot d\vec{r}$$

$$\int_i^f \vec{F}\cdot d\vec{r} = -\int_f^i \vec{F}\cdot d\vec{r}$$

$$\Rightarrow \; W_{i\to f} = -W_{f\to i}$$

Only along the same path.

Conservation of Energy -- pristine form



This is *always* correct:

Change in kinetic energy
= Total work done by the individual forces

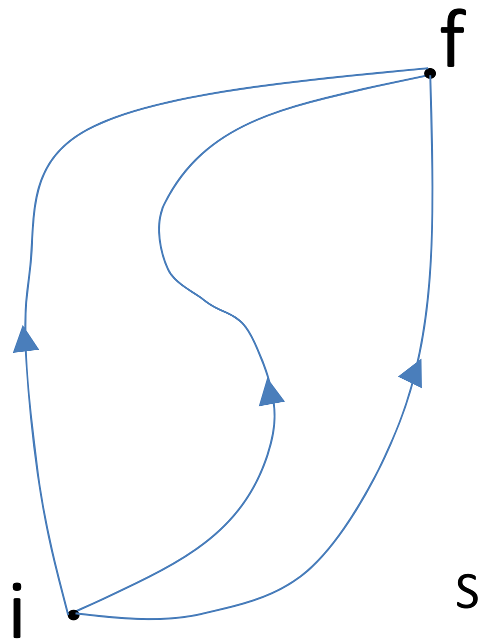
$$\Delta KE = (KE)_f - (KE)_i$$

$$= \sum_n \left(\int_i^f \vec{F}_n \cdot d\vec{r} \right)$$

$$\boxed{\Delta KE = \Sigma W_{i \to f}}$$

Conservative Force

If F is a function of position: $\vec{F}(\vec{r})$



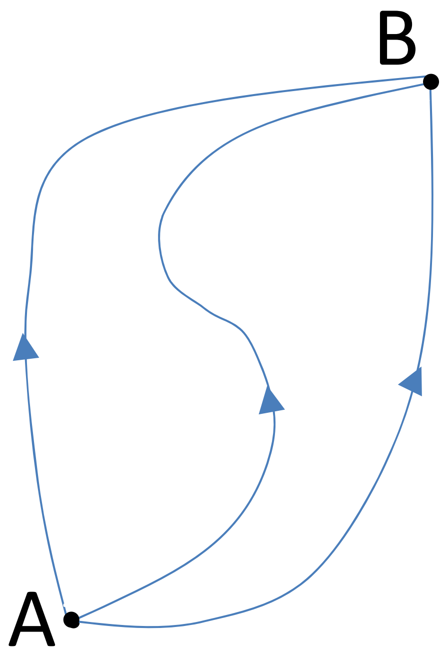
1. If $W_{i\to f}$ is path independent, F is conservative.

2. If $\vec{F}(\vec{r})$ is conservative, $W_{i\to i} = 0$ (why?)

Since the integral $W = \int \vec{F} \cdot d\vec{r}$ is path independent, we can "prepackage" the integration in general mathematical form.

Potential Energy U

If $\vec{F}(\vec{r})$ is conservative, the potential energy change ΔU is defined as the *negative* work done by the force $\vec{F}(\vec{r})$, which is path independent.



$$\Delta U = -\int_i^f \vec{F}(\vec{r}) \cdot d\vec{r}$$

Pay attention to the negative sign:
so we can move this term to the left hand side of the pristine conservation of energy equation $\Delta KE = \Sigma W_{i \to f}$ and becomes positive!

Conservation of Energy

Pristine form:

Change in kinetic energy = Total work done by the individual forces

$\Rightarrow$ Change in kinetic energy = Total work done by conservative forces + Total work done by non-conservative forces

$\Rightarrow$ $\Delta KE = -\Delta U +$ Total work done by non-conservative forces

$\Rightarrow$ $\Delta KE + \Delta U =$ Total work done by non-conservative forces

Secondary form

$$\Rightarrow \Delta KE + \Delta U = W_{\text{non-conservative forces}}$$

This is still *always* correct!

If there is no non-conservative forces:

$$\Delta KE + \Delta U = 0 \quad \Leftrightarrow \quad KE + U = \text{constant}$$

$$\Leftrightarrow \quad (KE)_i + U_i = (KE)_f + U_f$$

Close system form

This is correct only if there is no non-conservative force.