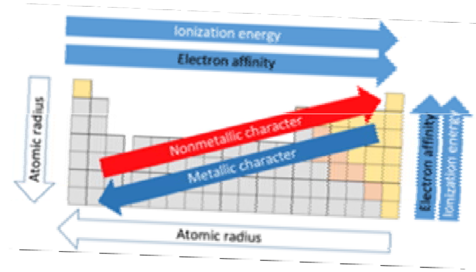
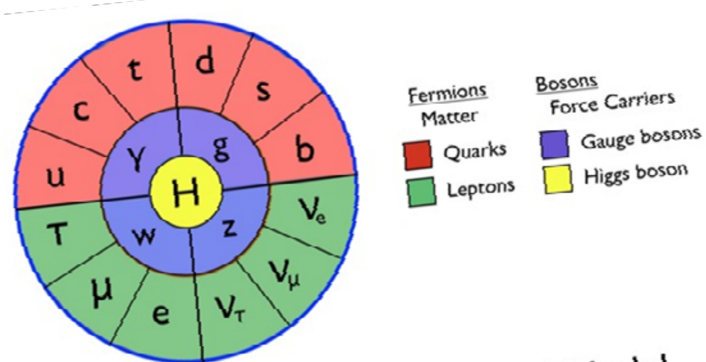
Currents

SPS Presents: A Cosmic Lunch!



- Who: Dr. Brown will be speaking about “Evolution of the Elements: from Periodic table to Standard Model and Beyond”!
- When: October 17th at 11am
- Where: CP-179 (by the front office)
- There will be pizza and soft drinks!



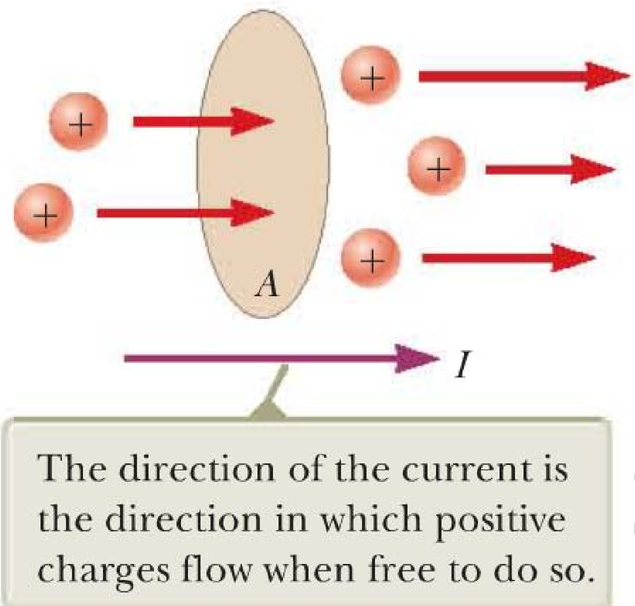
Particles of the Standard Model

Test 2 Next Wednesday (Oct 11)

1. Chapters 7, 8 (8.1-8.3).
2. You are not allowed to check your section number during the test. However, you will get 3 bonus points if you fill in your section number correctly.
3. 45 minutes sharp.
4. 4 multiple choices and 2 long problems.
5. Formula sheet provided.
6. Contact me before next Monday for prearrangement if you need special accommodation.

Currents

Current

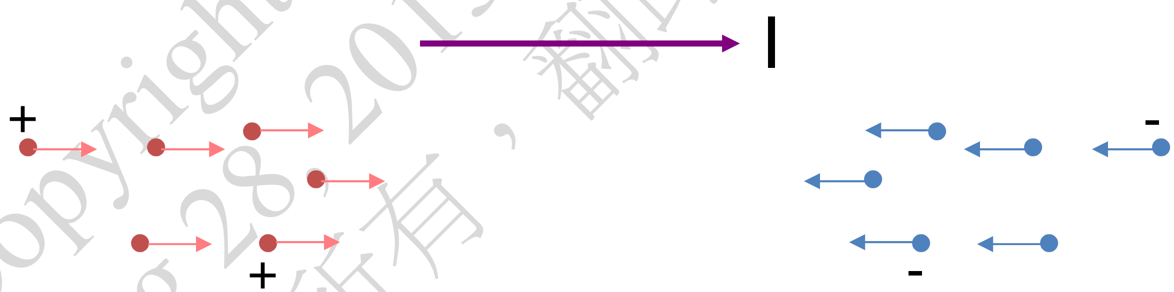


If dQ is the amount of charge passes through A in a short time interval dt, current is defined as:

$$I = \frac{dQ}{dt}$$

Units of current:

Ampere (A) ≡ C/s

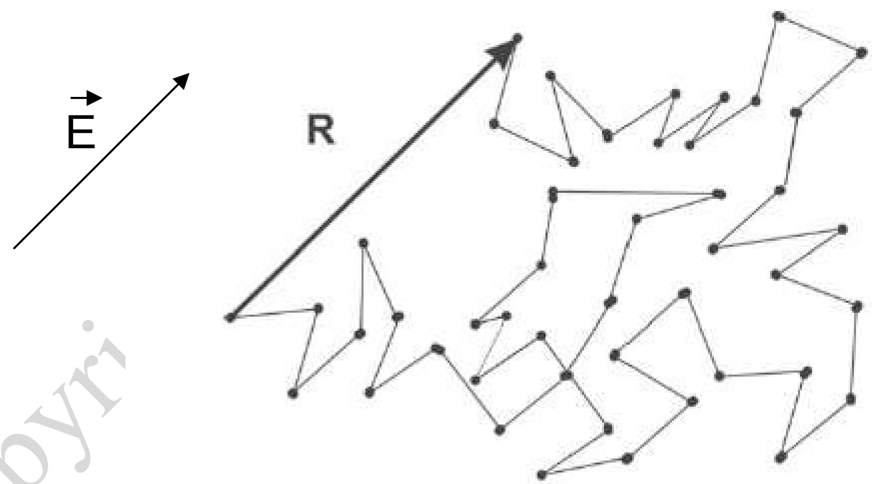


Electrically these two cases produce the same current, *but they can be distinguished with a magnetic field.*

Drifting velocity v_d

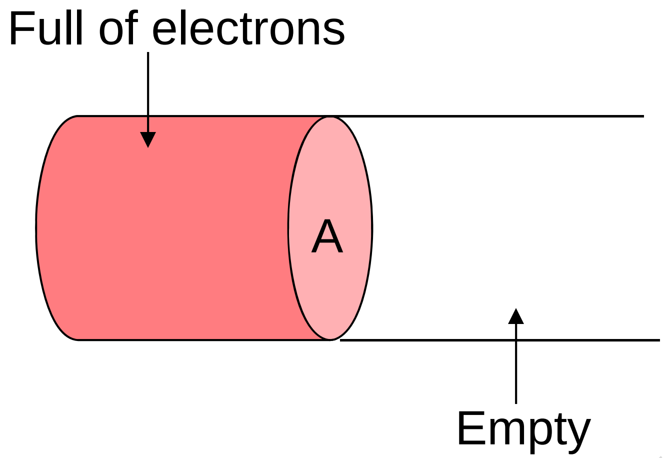
At any instant, electrons contributing to the current is moving very fast at about 10^6 m/s.

They also make collision with atoms and impurities very often, about 10^{14} times per second.

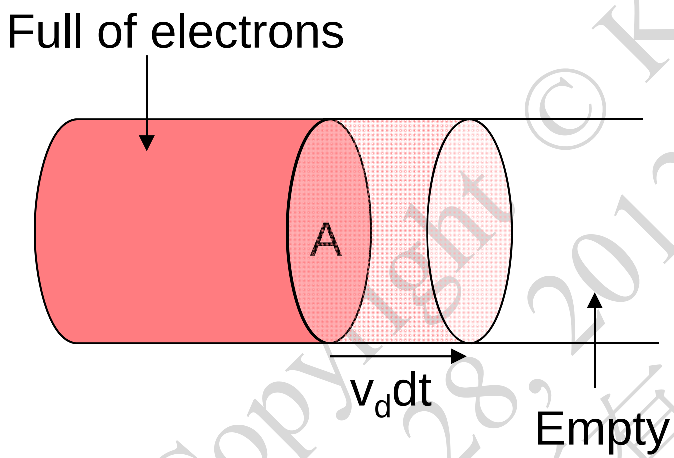


As a result, electrons drift very slowly along the electric field direction with a drifting velocity $v_d \sim 10^{-4}$ m/s.

Microscopic Model of Current



How many electron will pass the area A in a short time interval dt?



If n is the number of electrons per unit volume.

Number of electrons pass through area A = n×volume = $n(v_d dt)A$

If the charge of electron is e.

Charge pass through area A is

$$dQ = ne(v_d dt)A$$

$$\therefore I = \frac{dQ}{dt} \quad \Rightarrow \quad \boxed{I = nev_d A}$$

Class 20: Electric current and resistance, Ohm’s law

Drude's Model

Electrons collide with atoms. Let the velocity of an electron immediately after the collision be $\vec{v}_i$. Electrons will continue to accelerate in opposite direction to the electric field until the next collision.

$$\vec{v} = \vec{v}_i + \vec{a}\,t \Rightarrow \vec{v} = \vec{v}_i + \frac{e\vec{E}}{m} t$$

Drude's assumption: $\vec{v}_i$ is independent of the electron motion before the collision, and it has equal probability in pointing in any direction. i.e.

$$< \vec{v}_i > = 0$$

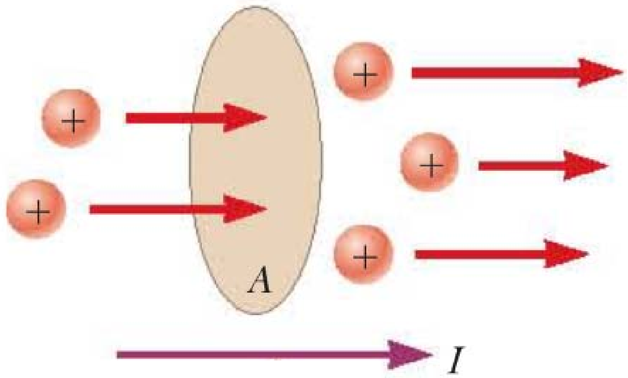
$$\therefore \vec{v} = \vec{v}_i + \frac{e\vec{E}}{m} t \Rightarrow < \vec{v} > = < \vec{v}_i > + \frac{e\vec{E}}{m} < t > \Rightarrow < \vec{v} > = \frac{e\vec{E}}{m} < t >$$

But $<\vec{v}> = \vec{v}_d$, and let $<t> = \tau$, average time between collisions.

$$\therefore \vec{v}_d = \frac{e\vec{E}}{m}\tau \Rightarrow \vec{j} = ne\,\vec{v}_d = \frac{ne^2\tau}{m}\vec{E}$$

$$\Rightarrow \vec{j} = \sigma\,\vec{E} \quad \text{if } \sigma = \frac{ne^2\tau}{m}$$

Current Density and Ohm's Law (physics version)



Current density

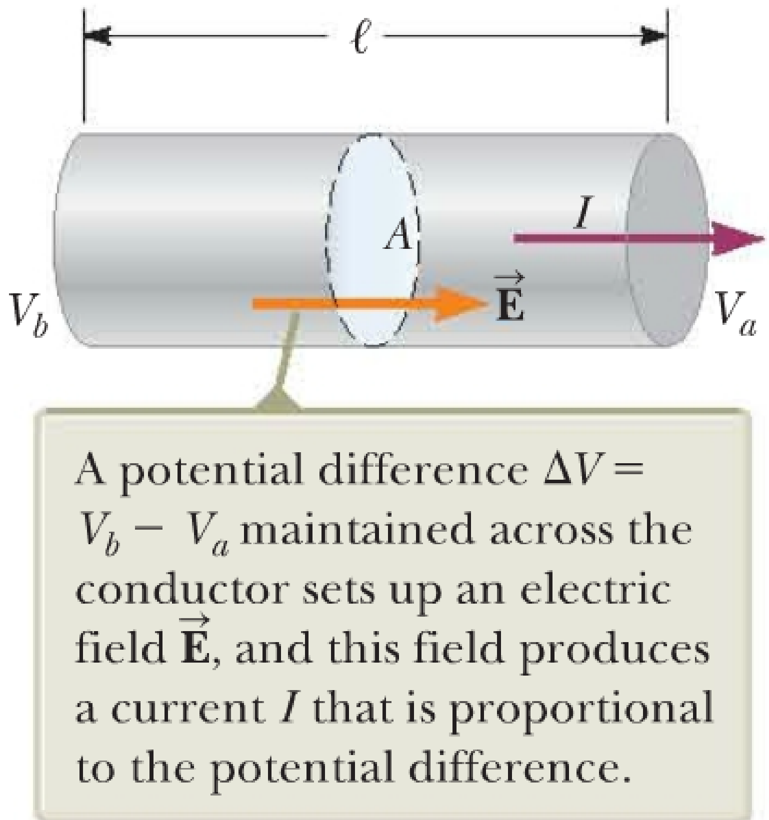
$$J = \frac{I}{A}$$

Ohm's Law (physics version)

$$\vec{J} \propto \vec{E} \;\Rightarrow\; \boxed{\vec{J} = \sigma \vec{E}} \qquad \text{or} \qquad \vec{J} = \frac{1}{\rho}\vec{E}$$

1. σ is called conductivity. *Do not confuse this with the surface charge density.*
2. ρ is called resistivity. *Do not confuse this with the volumetric charge density.*
3. σ and ρ represent the same information, $\boxed{\rho = \frac{1}{\sigma}}$.
4. σ and ρ are properties of materials.

Ohm's Law (electronics version)



$J \to I$ and $E \to V$

Ohm's Law:

$$J = \frac{1}{\rho} E \;\Rightarrow\; \frac{I}{A} = \frac{1}{\rho} \cdot \frac{\Delta V}{\ell}$$

$$\Rightarrow \Delta V = \left(\frac{\rho \, \ell}{A} \right) I$$

$$\Rightarrow \boxed{\Delta V = I R}$$

where

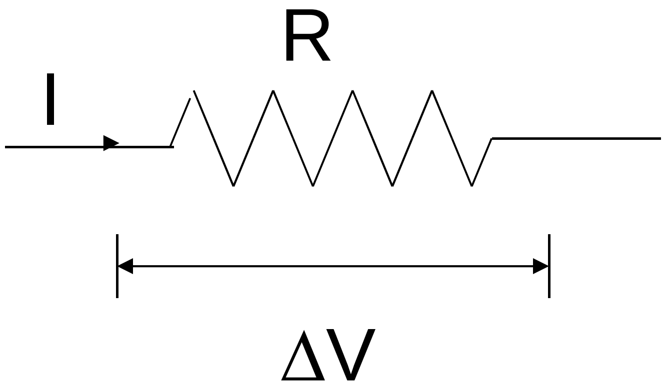
$$\boxed{R = \frac{\rho \, \ell}{A}}$$

1. R is called the resistance.
2. Units of resistance is Ohm (Ω). $\Omega \equiv V/A$

Power

Power dissipated in resistance R:

$$P = I\Delta V = I^2R = \frac{\Delta V^2}{R}$$



Units of power:
Watt (W) $\equiv$ J/s

Resistance, Conductance, Resistivity, and Conductivity

Ohm's Law:

Resistivity ρ
Units: Ωm

$$\vec{J} = \frac{1}{\rho}\vec{E}$$

$$\rho = \frac{1}{\sigma}$$

$$\vec{J} = \sigma\,\vec{E}$$

Conductivity σ
Units: S/m

$$R = \frac{\rho\,\ell}{A}$$

$$G = \frac{A\,\sigma}{\ell}$$

$$R = \frac{1}{G}$$

Resistance R
Units: Ω

$$\Delta V = I\,R$$

$$\Delta V = \frac{I}{G}$$

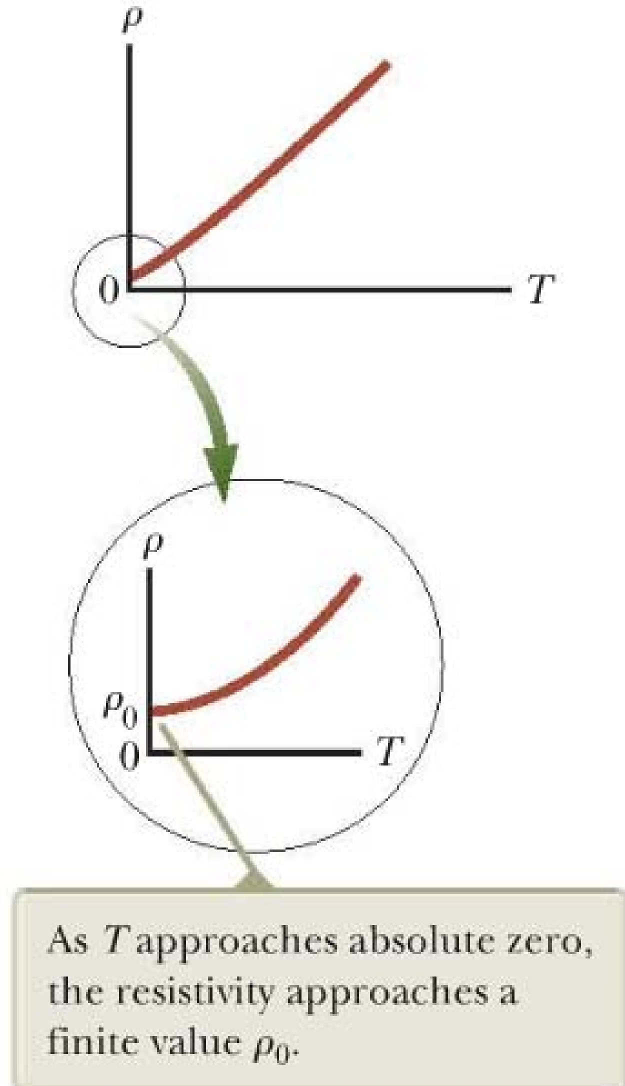
Conductance G
Units: Siemens (S)
or mho

1. Resistivity and conductivity depend on the materials only, Resistance and conductance depend on both the materials and also the geometry of the conductor.
2. Ohm's Law: ΔV = IR or J = σE
3. $R = \frac{\rho\,\ell}{A}$
4. $\rho = \frac{1}{\sigma}$ and $R = \frac{1}{G}$

Resistance and Temperature

For a normal conductor:

As temperature rises, electrons will make collisions more frequently. As a result, τ decreases and hence ρ increases.



Linear approximation:

$$\frac{\rho - \rho_0}{T - T_0} = \alpha\, \rho_0 \;\Rightarrow\; \rho = \rho_0[1 + \alpha\,(T - T_0)]$$

$$\text{or } R = R_0[1 + \alpha\,(T - T_0)]$$

α=Temperature coefficient of resistivity

T_0 = 20^{o}C