Kirchhoff’s Rules

Kirchhoff's Voltage Rule – sign convention

1. Use only currents as unknown variables. ΔV can always be written in terms of currents or derivatives or integrals of currents.

2. Assign current direction to every path in a circuit. Apply Kirchhoff's current rule as much as you can to reduced the number of unknown variables.

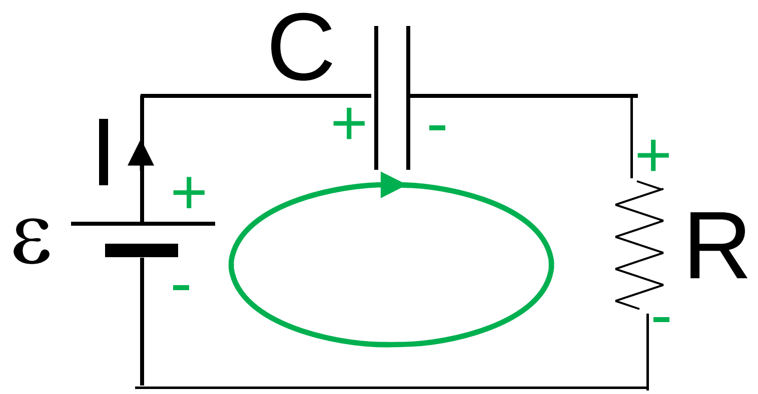
3. Across every component in the circuit determine which end has a higher potential (mark it with a + sign) and which end has a lower potential (mark it with a - sign). This will depend on the current direction you assume in step 2.

4. Pick up a loop and travel around it either clockwise or anticlockwise. If you travel from – to + across a component, then ΔV across the component is positive. If you travel from + to – across a component, then ΔV across the component is negative. You can reverse this convention as long as you do it consistently for the whole loop.

5. If you get a negative current, that means the current direction you assume in step 2 is wrong and you should reverse that direction.

Kirchhoff's Voltage Rule – sign convention

Example:

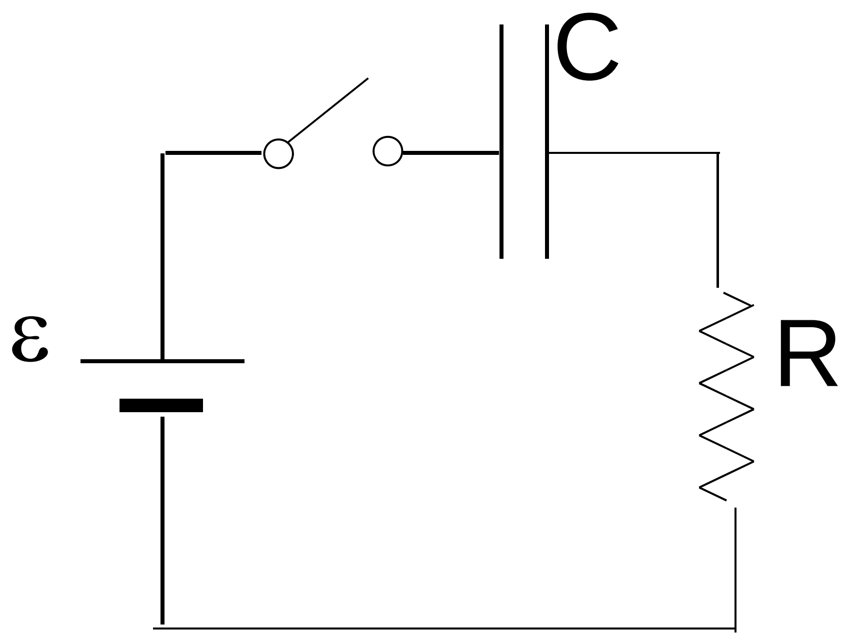


$$-\underbrace{\frac{Q}{C}}_{\Delta V_C}-\underbrace{IR}_{\Delta V_R}+\underbrace{\varepsilon}_{\Delta V_{bat}}=0$$

As you travel around a loop, if you find yourself moving from + to - , make ΔV across that component negative (C and R in this example).

Class 25: RC Circuits

RC Circuits – Charging

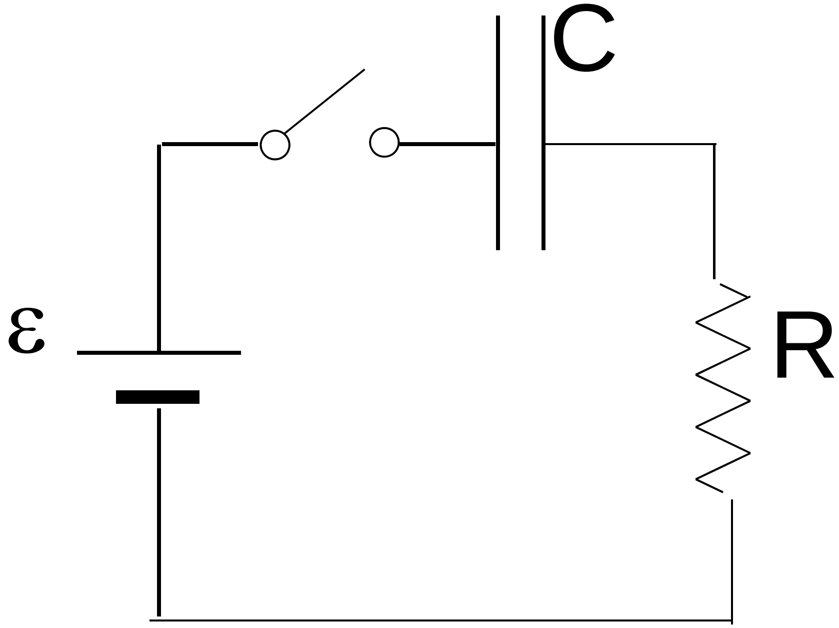


At t=0, capacitance is uncharged and Q=0 (initial condition).

At t=0, switched is closed, it the capacitor has no charge, it behaves like a conductor and I=ε/R.

After the capacitor is completely charged, Q=C ε, $\Delta V_C = \varepsilon$ and $\Delta V_R = 0$. I=0 and the capacitors behave like an insulator.

RC Circuits – Charging



$$\varepsilon = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = \varepsilon$$

$$\Rightarrow CR\,dq = (C\varepsilon - q)\,dt$$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n(q - C\varepsilon) = -\frac{t}{CR} + K'$$

$$\Rightarrow q - C\varepsilon = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = C\varepsilon + K e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = 0 \Rightarrow 0 = C\varepsilon + K \Rightarrow K = -C\varepsilon$$

$$\therefore q = \underline{\underline{C\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$$I = \frac{dq}{dt} = \frac{C\varepsilon}{CR}e^{-\frac{t}{CR}} = \underline{\underline{\frac{\varepsilon}{R}e^{-\frac{t}{CR}}}}$$

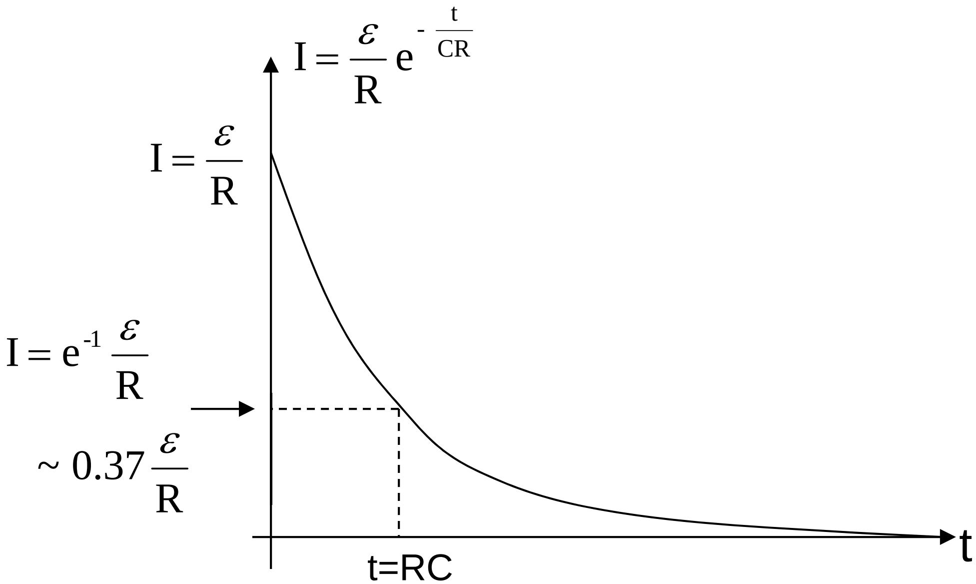
$$\Delta V_R = IR = \underline{\underline{\varepsilon e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$\Delta V_R + \Delta V_C = \varepsilon$

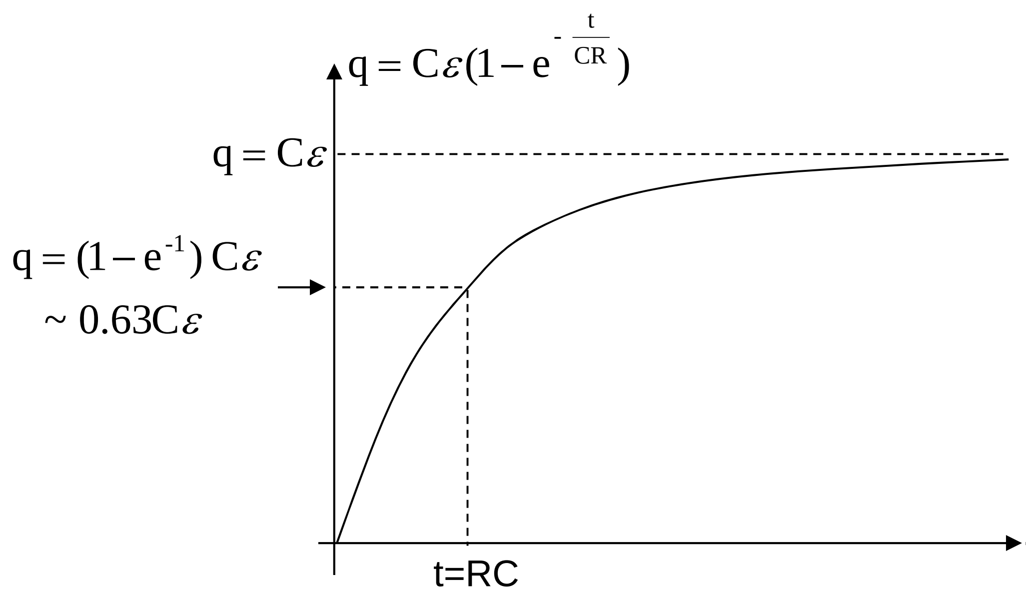
RC time constant

τ=RC is known as the RC time constant. It indicates the response time (how fast you can charge up the capacitor) of the RC circuit.



$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$

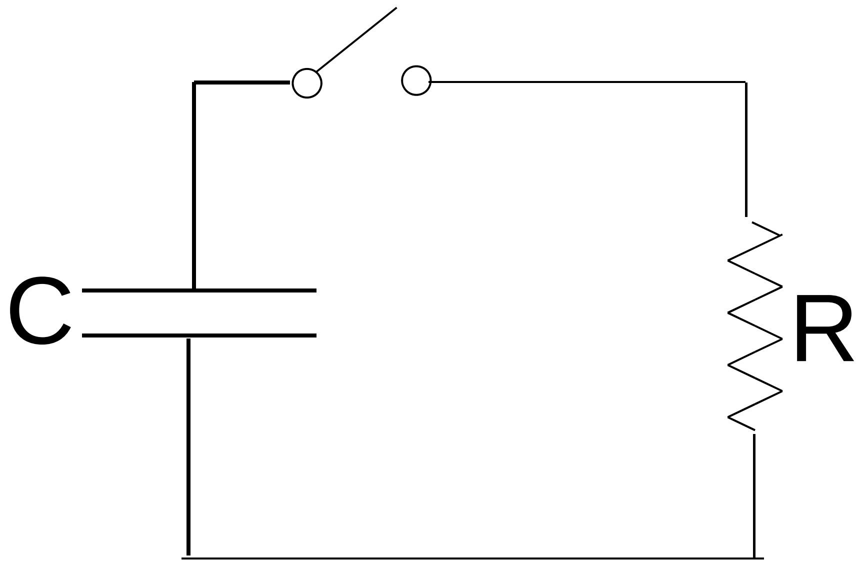


$$\sqrt{2} \approx 1.414$$

$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RC circuits

RC Circuits – Discharging

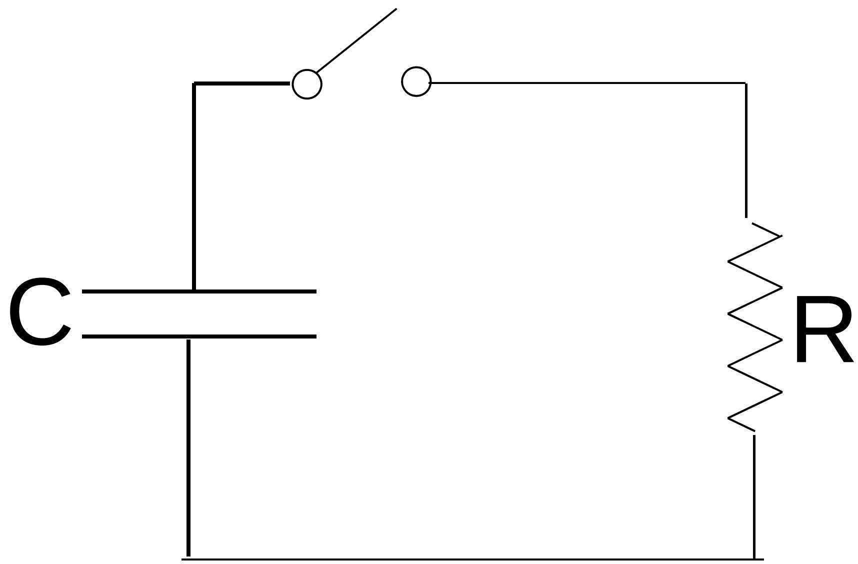


At t=0, capacitance is charged with a charge Q (initial condition).

At t=0, switched is closed, the capacitor starts to discharge.

After the capacitor is completely discharged, Q=0, ΔV_C= 0, ΔV_R=0 and I=0.

RC Circuits – Discharging



$$0 = \frac{q}{C} - IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = 0 \qquad (I = -\frac{dq}{dt})$$

$$\Rightarrow CR\,dq = -q\,dt$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n\, q = -\frac{t}{CR} + K'$$

$$\Rightarrow q = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = K\,e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = Q \Rightarrow Q = K$$

$$\therefore q = \underline{\underline{Qe^{-\frac{t}{CR}}}}$$

$$I = \frac{dq}{dt} = -\underline{\underline{\frac{Q}{RC}e^{-\frac{t}{CR}}}}$$

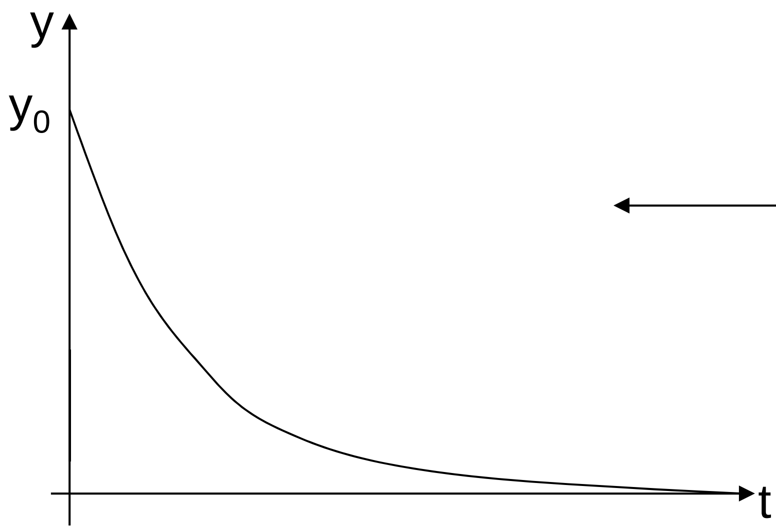
$$\Delta V_R = IR = -\underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$\Delta V_R + \Delta V_C = 0$

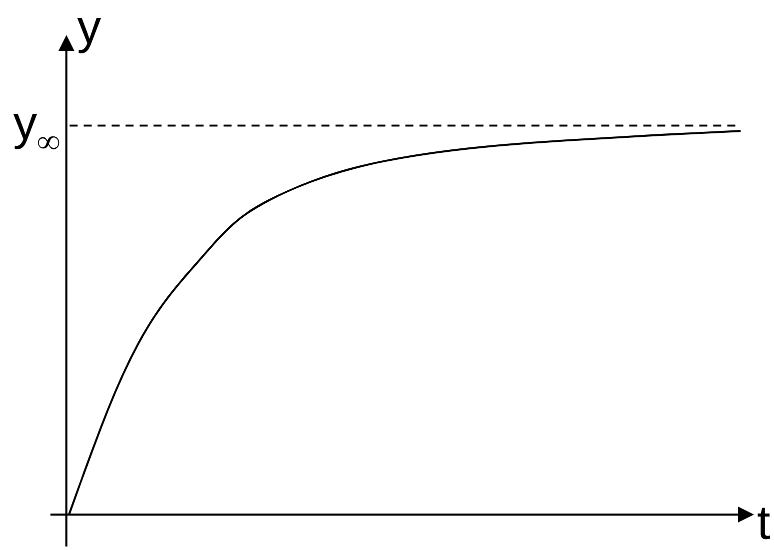
In Summary

For both charge and discharge, Q, I, ΔV_C, and ΔV_R must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

$$y = y_\infty (1 - e^{-\frac{t}{RC}})$$



y can be Q, I, ΔV_C, or ΔV_R