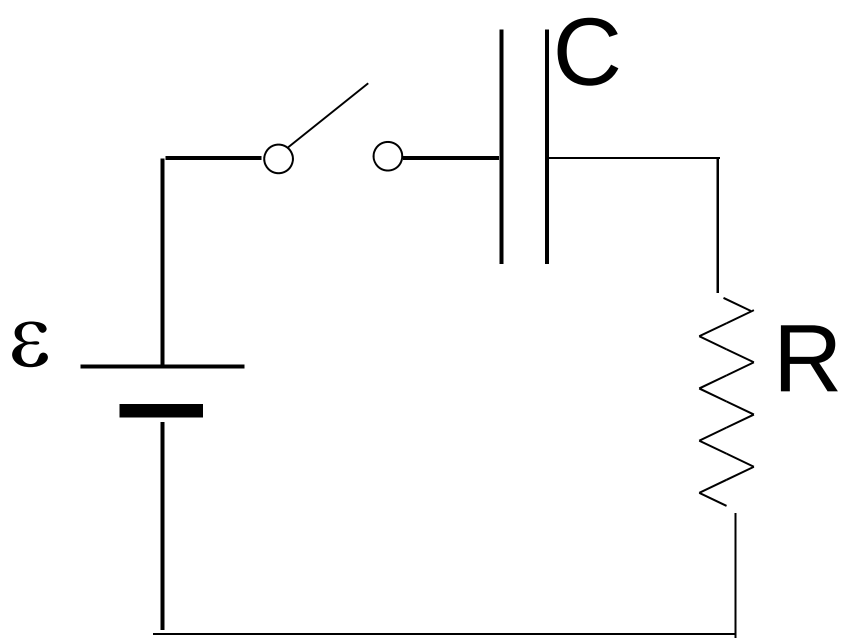
RC Circuits

RC Circuits – Charging

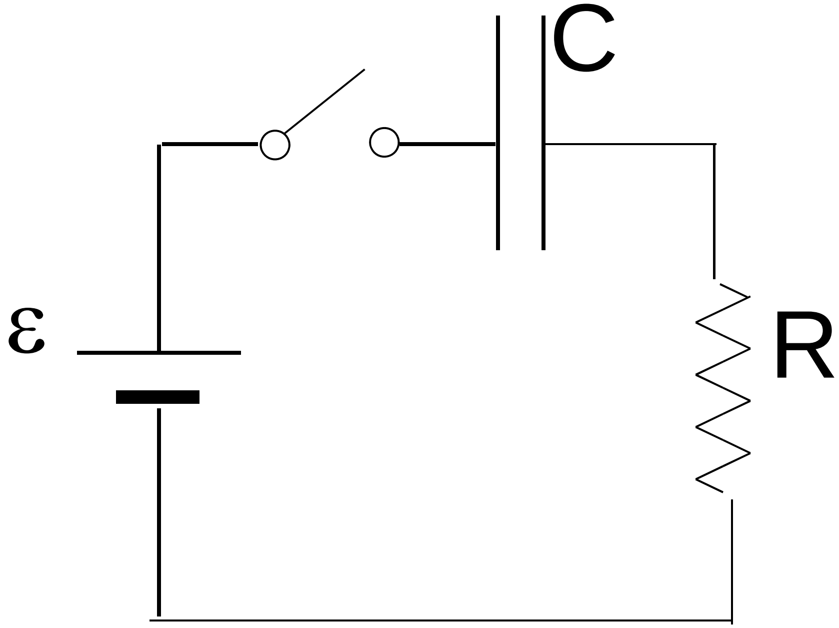


At t=0, capacitance is uncharged and Q=0 (initial condition).

At t=0, switched is closed, it the capacitor has no charge, it behaves like a conductor and I=ε/R.

After the capacitor is completely charged, Q=C ε, ΔV_C= ε and ΔV_R=0. I=0 and the capacitors behave like an insulator.

RC Circuits – Charging



$$\varepsilon = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = \varepsilon$$

$$\Rightarrow CR\,dq = (C\varepsilon - q)\,dt$$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n(q - C\varepsilon) = -\frac{t}{CR} + K'$$

$$\Rightarrow q - C\varepsilon = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = C\varepsilon + K e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = 0 \Rightarrow 0 = C\varepsilon + K \Rightarrow K = -C\varepsilon$$

$$\therefore q = \underline{\underline{C\varepsilon(1 - e^{-\frac{t}{CR}})}}$$

$$I = \frac{dq}{dt} = \frac{C\varepsilon}{CR}e^{-\frac{t}{CR}} = \underline{\underline{\frac{\varepsilon}{R}e^{-\frac{t}{CR}}}}$$

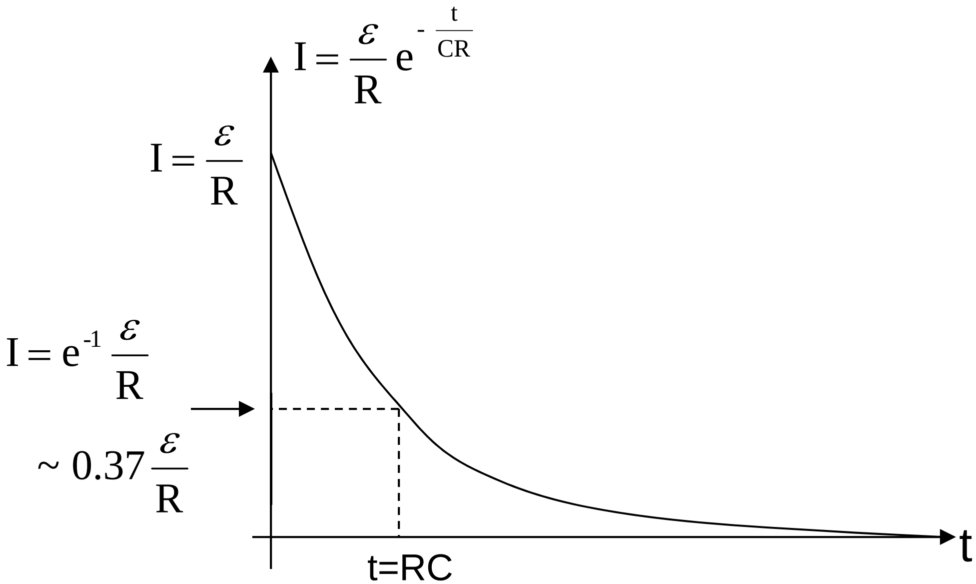
$$\Delta V_R = IR = \underline{\underline{\varepsilon\, e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\varepsilon\,(1 - e^{-\frac{t}{CR}})}}$$

$\Delta V_R + \Delta V_C = \varepsilon$

RC time constant

$\tau = RC$ is known as the RC time constant. It indicates the response time (how fast you can charge up the capacitor) of the RC circuit.



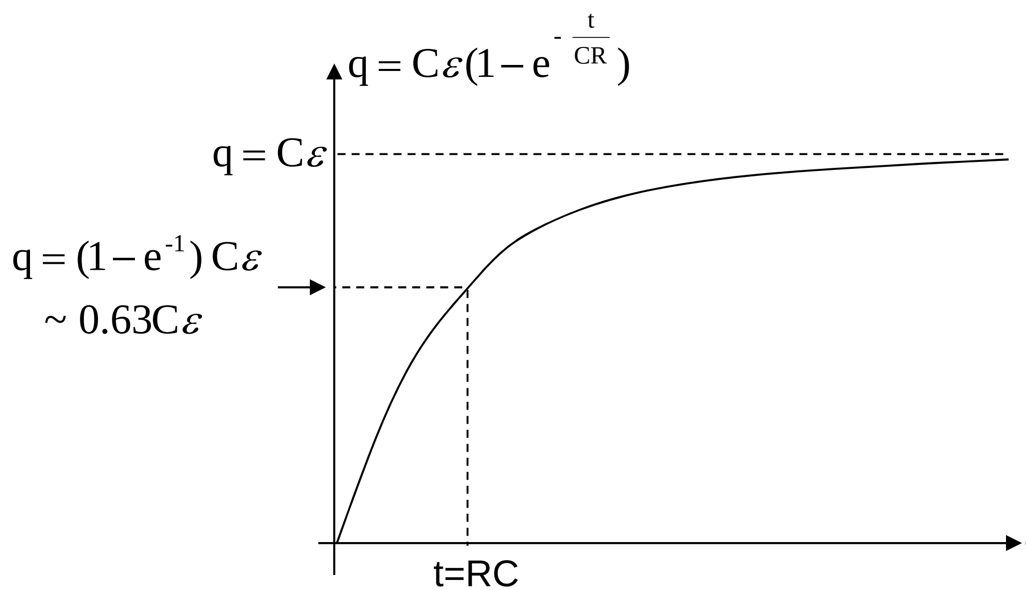
$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$

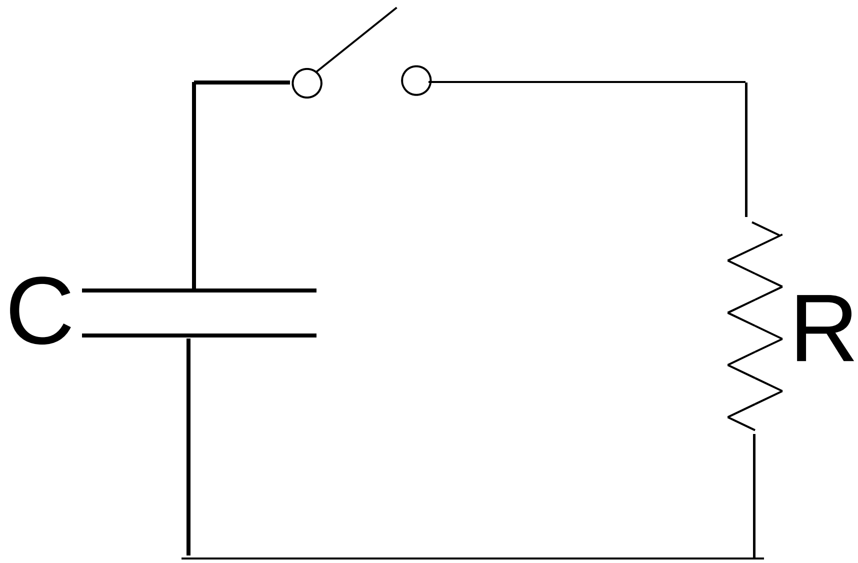
$$\sqrt{2} \approx 1.414$$

$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RC circuits



RC Circuits – Discharging

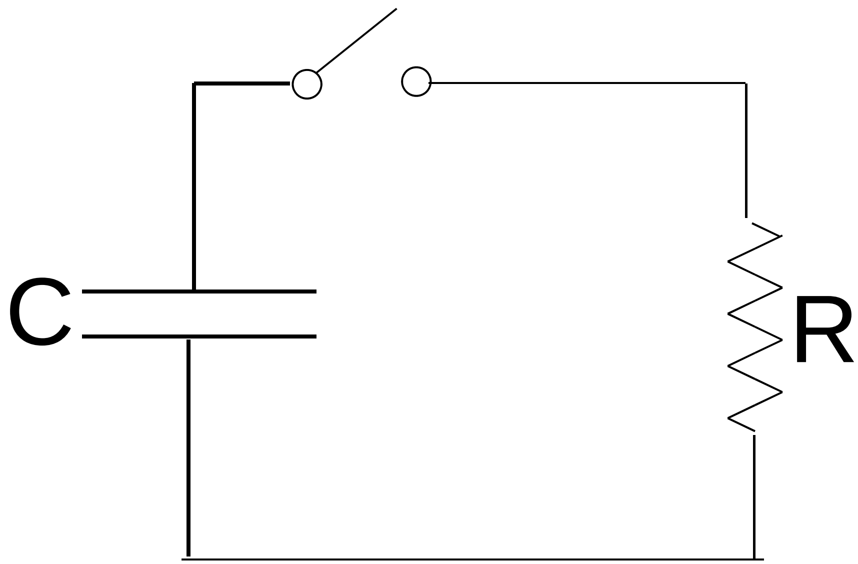


At t=0, capacitance is charged with a charge Q (initial condition).

At t=0, switched is closed, the capacitor starts to discharge.

After the capacitor is completely discharged, Q=0, ΔV_C= 0, ΔV_R=0 and I=0.

RC Circuits – Discharging



$$0 = \frac{q}{C} - IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = 0 \qquad (I = -\frac{dq}{dt})$$

$$\Rightarrow CR\,dq = -q\,dt$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n\, q = -\frac{t}{CR} + K'$$

$$\Rightarrow q = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = K\,e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = Q \Rightarrow Q = K$$

$$\therefore\ q = \underline{\underline{Qe^{-\frac{t}{CR}}}}$$

$$I = \frac{dq}{dt} = -\underline{\underline{\frac{Q}{RC}e^{-\frac{t}{CR}}}}$$

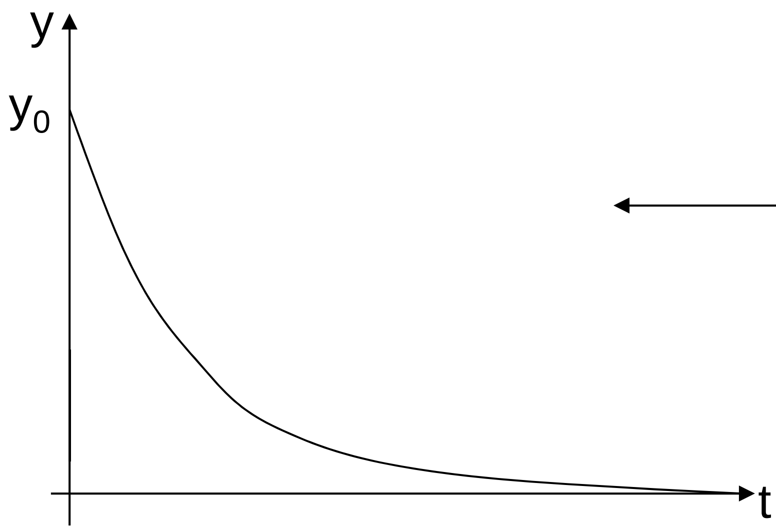
$$\Delta V_R = IR = -\underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$\Delta V_R + \Delta V_C = 0$

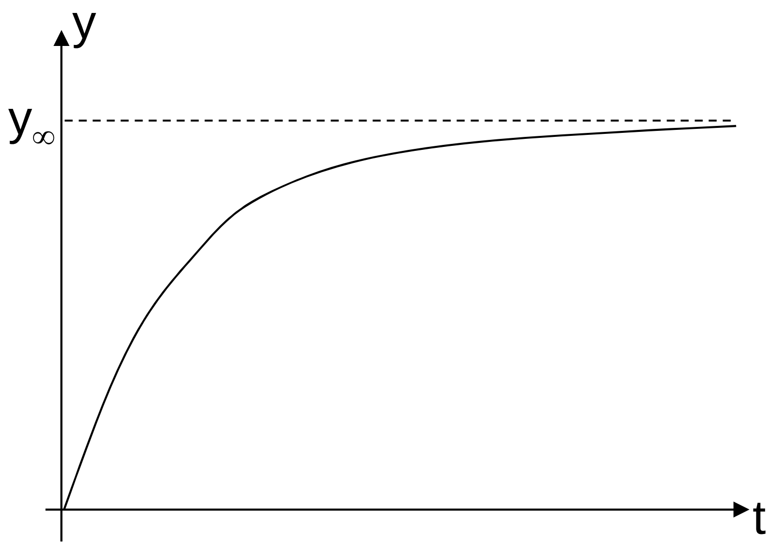
In Summary

For both charge and discharge, Q, I, ΔV_C, and ΔV_R must be one of the following two cases:



$$y = y_0 e^{-\frac{t}{RC}}$$

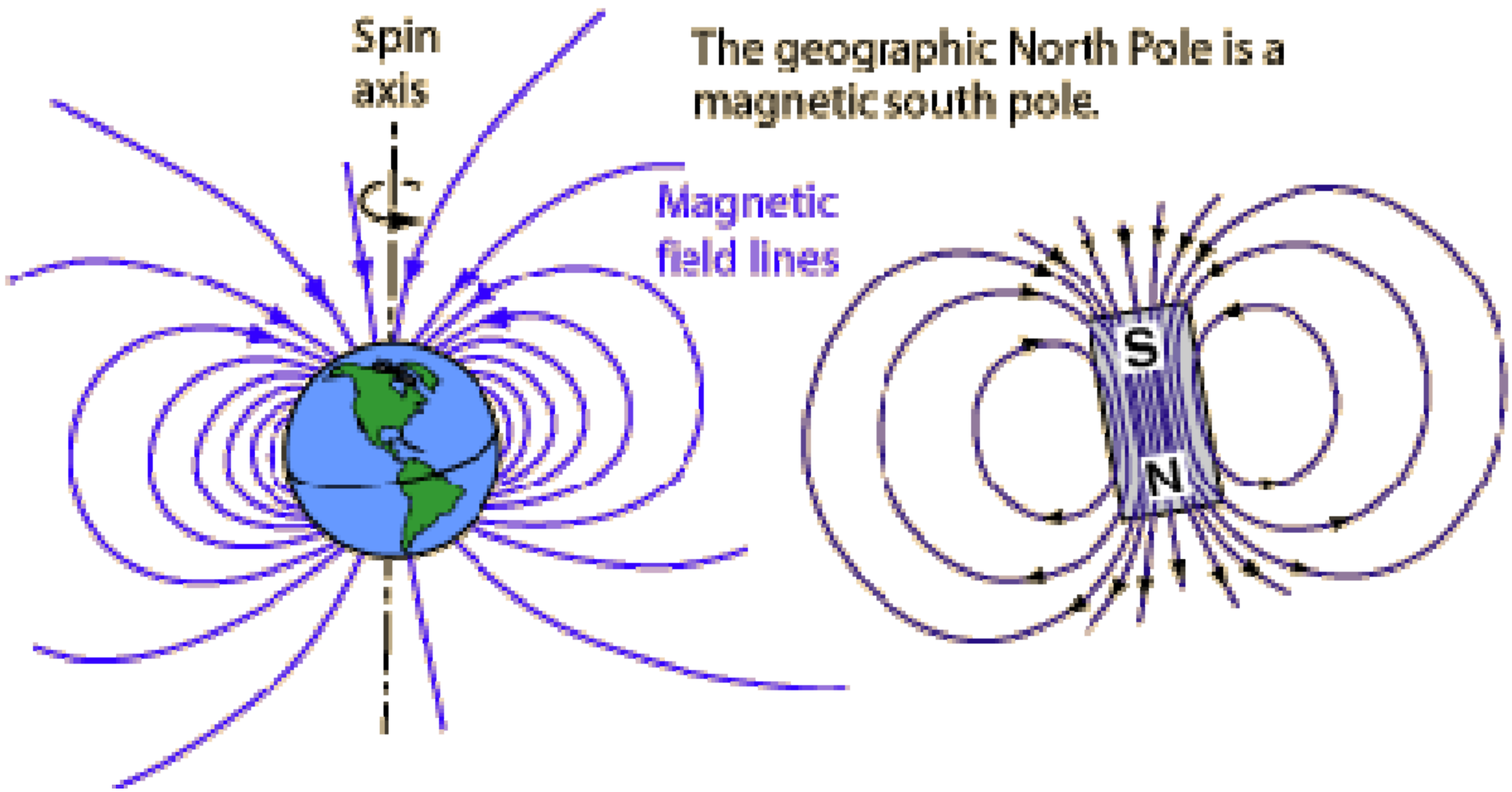
$$y = y_\infty (1 - e^{-\frac{t}{RC}})$$



y can be Q, I, ΔV_C, or ΔV_R

Class 26: Magnetic force acting on a moving point charge

Magnetic Field

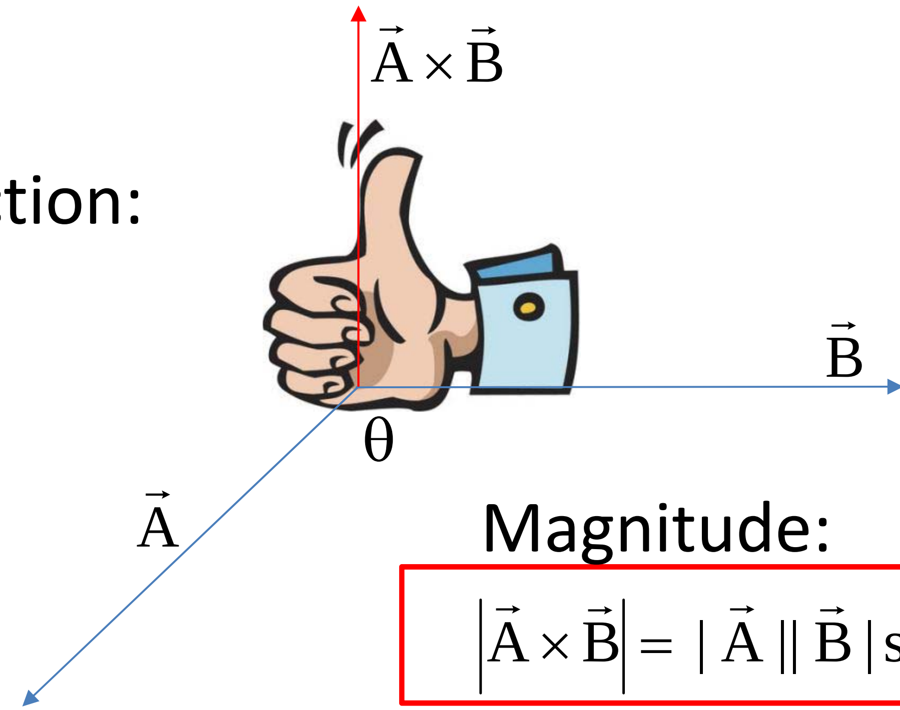


1. All single magnets have two poles, N and S.
2. Externally, magnetic field lines come out from the N pole and getting into the S pole.
3. Between two magnets, like poles repel and unlike poles attract.
4. The geographical north pole of earth is actually the S pole of a bar magnet.
5. We will explain why there is magnetic field later.

From class 3

cross product between two vectors

Direction:



Magnitude:

$$\left|\vec{A}\times\vec{B}\right| = |\vec{A}||\vec{B}|\sin\ \theta$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

From class 3

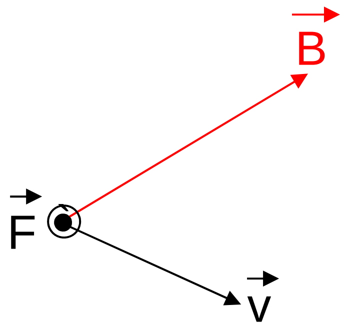
A common symbol

$\otimes$ or $\times$ A vector perpendicular and pointing into the screen /paper.

$\odot$ or $\bullet$ A vector perpendicular and pointing out of the screen /paper.

Magnetic Force Acting on a Moving Charge

When a charge particle moves in a magnetic field B, there will be magnetic force acting on the particle:



$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

1. Unit of magnetic field is Tesla (T).
2. If there is magnetic field, only under two conditions the magnetic force on the charge particle will be zero: (i) the particle is not moving (v=0), or (ii) it is moving in parallel or antiparallel to the magnetic field ($\sin\theta=0$).
3. The magnetic force is always perpendicular to the magnetic field and the velocity.
4. The magnetic force does no work because $\vec{F}_B \cdot \vec{v} = 0$.
5. If you want to determine the direction acting on a negative charge particle, treat it like a positive charge first, then reverse the force direction at the end.