

Energy and Conservative Law

Conservation of Energy

Pristine form:

Change in kinetic energy = Total work done by the individual forces

⇒ Change in kinetic energy = Total work done by conservative forces +
Total work done by non-conservative forces

⇒ $\Delta KE = -\Delta U + \text{Total work done by non-conservative forces}$

⇒ $\Delta KE + \Delta U = \text{Total work done by non-conservative forces}$

Secondary
form

⇒ $\Delta KE + \Delta U = W_{\text{non-conservative forces}}$

This is still *always* correct!

If there is no non-conservative forces:

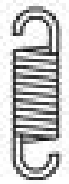
$$\begin{aligned}\Delta KE + \Delta U = 0 &\Leftrightarrow KE + U = \text{constant} \\ &\Leftrightarrow (KE)_i + U_i = (KE)_f + U_f\end{aligned}$$

Close system
form

This is correct only if there is no non-conservative force.

Three common conservative forces

Spring



Hooke's Law:

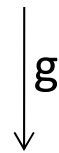
$$F = -kx$$

$$U = \frac{1}{2} kx^2$$

(U at natural length = 0)

Earth surface

y —————



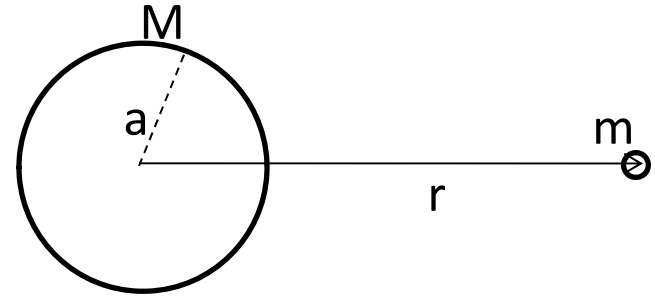
y = 0 —————

$$F = -mg$$

$$U = mgy$$

(U = 0 at y = 0)

Uniform spherical object



For $r > a$ only:

$$|F(r)| = G \frac{Mm}{r^2}$$

$$U(r) = -G \frac{Mm}{r}$$

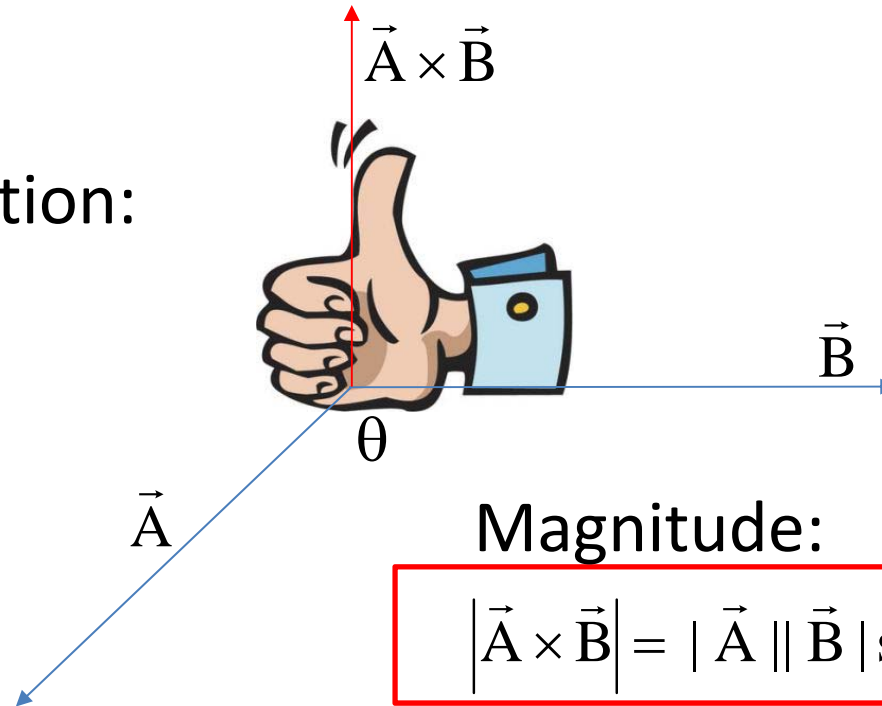
$$U(\infty) = 0$$

Class 3: Torque and rotational motion

cross product between two vectors



Direction:



Magnitude:

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

A common symbol



or



A vector perpendicular and pointing into the screen /paper.

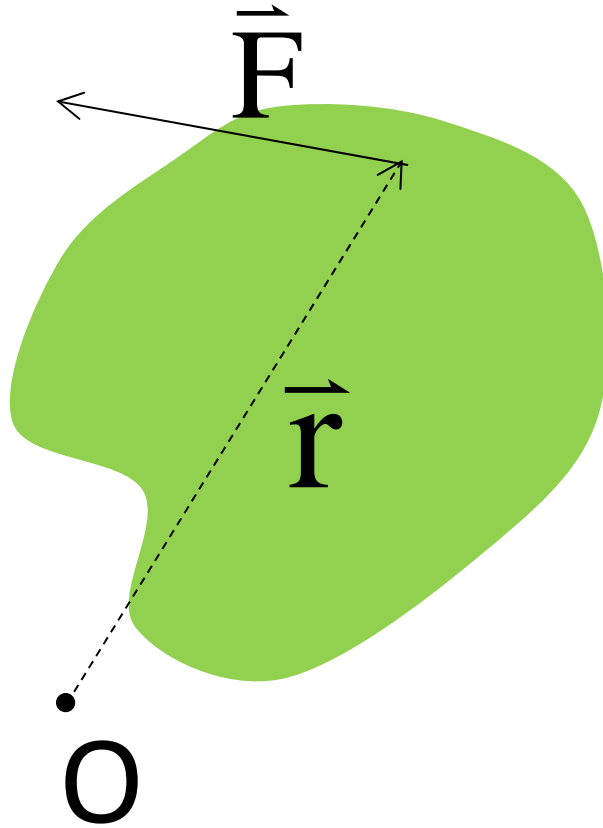


or



A vector perpendicular and pointing out of the screen /paper.

Torque



$$\tau = \vec{r} \times \vec{F}$$

Torque depends on:

(i) at what point of the rigid body the force is acting on.

(ii) how we choose the origin.