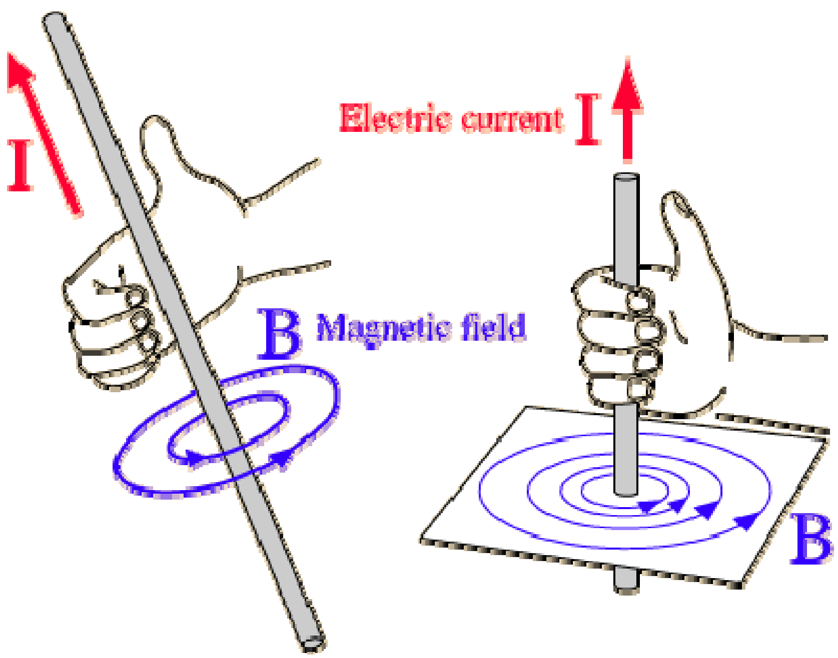
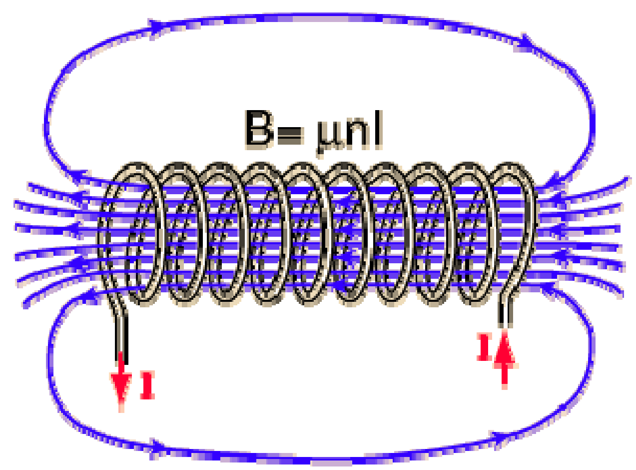
Origin of Magnetic field

A current (or moving charge) experience a magnetic force when it is in a magnetic field. The magnetic field is the result of another current (or moving charge).
If electric field describes the interaction between two charges, then magnetic field describes the interaction between two currents (or moving charges).



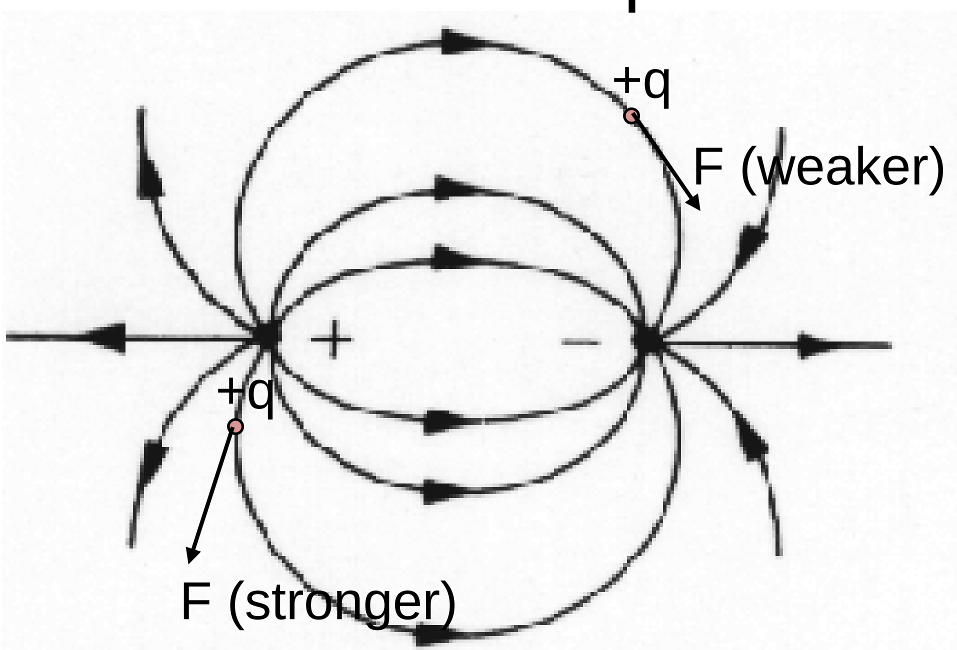
Magnetic field due to a long current



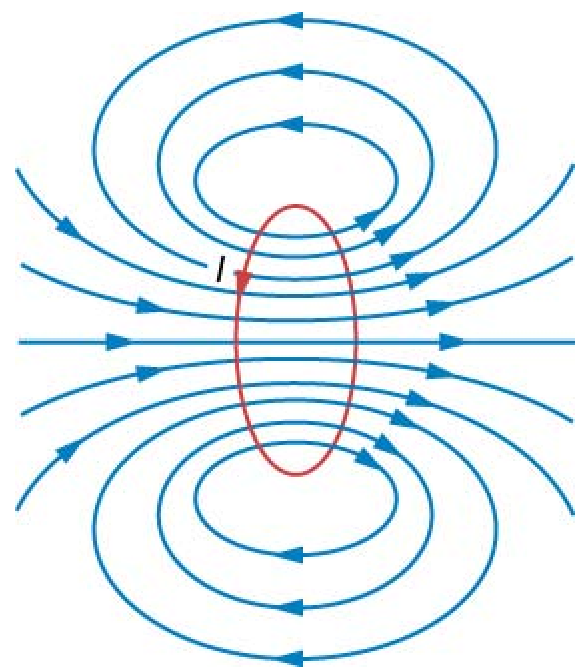
Magnetic field of a solenoid

Properties of field lines I

1. To visualize the ~~electric field~~ magnetic field, we draw field lines. When we put a ~~*positive* test charge~~ Small bar magnet in the ~~electric field,~~ magnetic field the force acting on it will be tangent to the field line at that point. The magnitude of the force will be proportional to the density of field lines at that point.



electric field



magnetic field

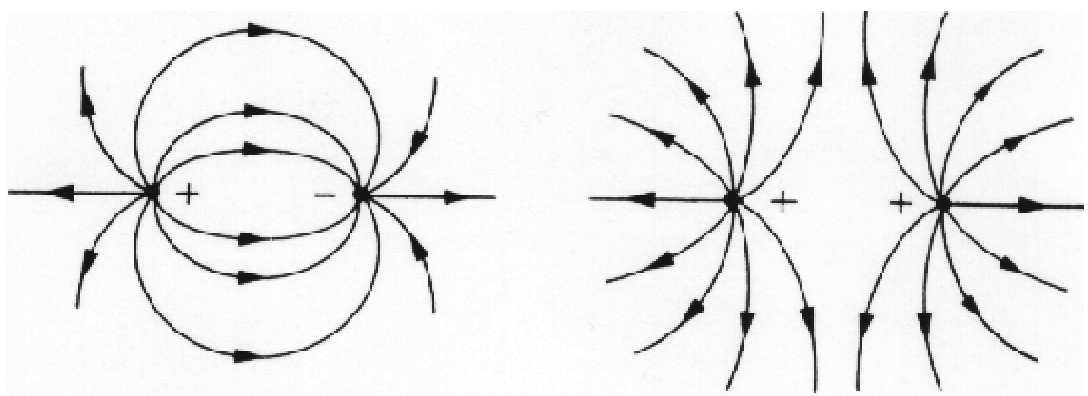
Properties of field lines II

magnetic field

2. ~~Electric field~~ lines are continuous lines only terminate ~~at charges or~~ at infinity.

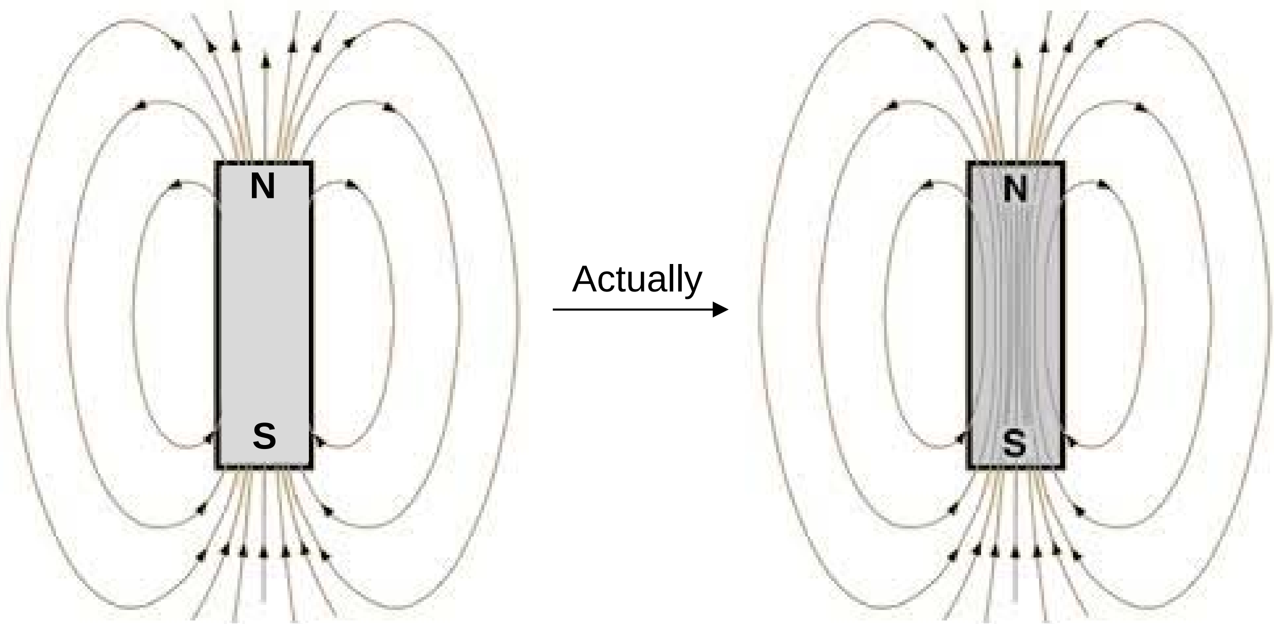
3. ~~When an electric field line terminate at charges, it always comes out from a positive charge, or getting into a negative charge.~~

4. Field lines never cross each other.



Consequences of non-existence of magnetic charge (monopole)

1. Field lines terminate at point charges, so magnetic field lines either terminate at infinity, or form loops.



2. Gauss's Law:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}$$

Electric field

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Magnetic field

Maxwell's Equations

Maxwell's equations describe all the properties of electric and magnetic fields and there are four equations in it:

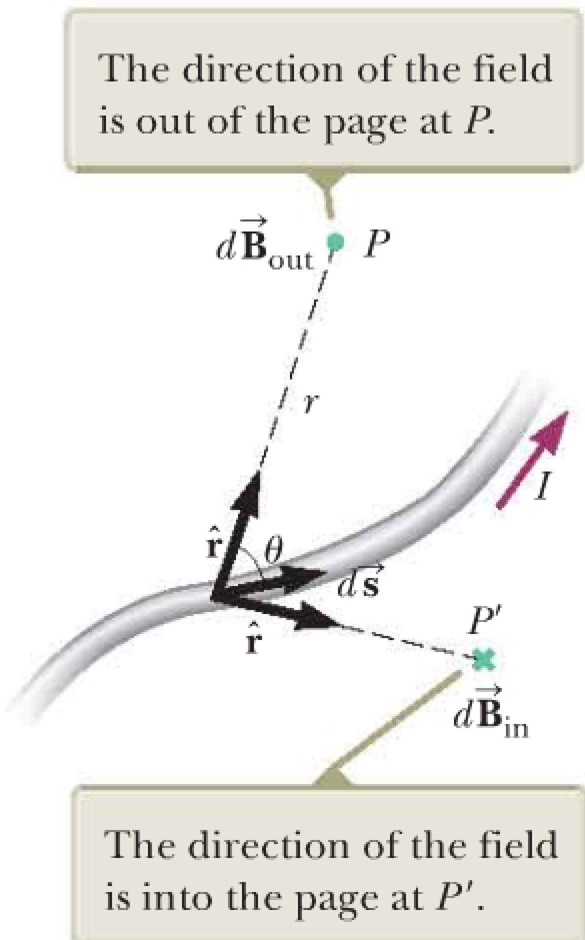
	Integral form	Differential form (optional)	Name of equation
1st Equation	$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{enclosed}$	$\varepsilon_0 \nabla \cdot \vec{E} = \rho$	Electric Gauss's Law
2nd Equation	$\oint \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$	Magnetic Gauss's Law
3rd Equation	Not yet		
4th Equation	Not yet		

Lorentz force equation is not part of Maxwell's equations. It describes what happens when charges are put in an electric or magnetic fields:

$$\vec{F} = (q\vec{E} + \vec{v} \times \vec{B})$$

Class 32. Biot-Savart Law

Biot-Savart Law



Magnetic field at point P due to the infinitesimal element ds:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} \quad \text{or} \quad \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\vec{r}}{r^3}$$

Magnetic field due to the whole wire:

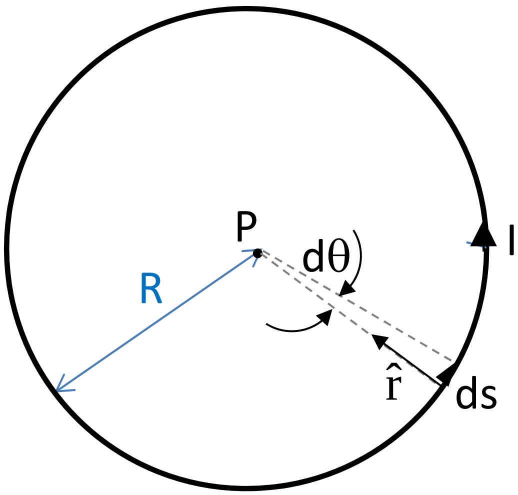
$$\vec{B} = \int_{\text{wire}} \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$$

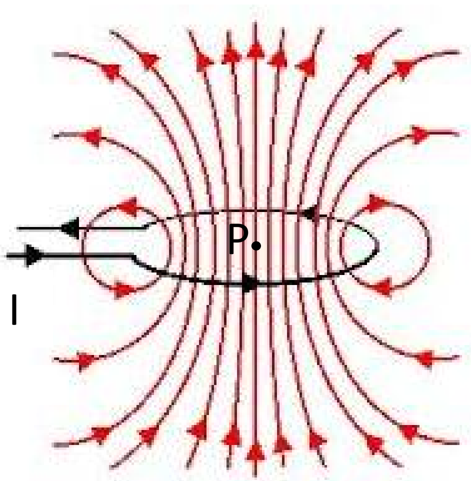
μ_0 is a constant called permeability of free space:

$\mu_0 = 4\pi\times10^{-7}\ \text{TmA}^{-1}$

In the calculation of magnetic field, Biot-Savart Law play the same role as the Coulomb's Law in electric field.

Magnetic Field at the Center of a Circular Current Loop





$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{ds}{R^2} \odot$$

$$ds = R d\theta$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{R d\theta}{R^2} \odot$$

$$= \frac{\mu_0 I}{4\pi R} d\theta \odot$$

$$\therefore \ \vec{B} = \int_0^{2\pi} \frac{\mu_0 I}{4\pi R} d\theta \odot = \odot \frac{\mu_0 I}{4\pi R} \int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R}[\theta]_0^{2\pi} \odot$$

$$= \frac{\mu_0 I}{4\pi R} \cdot 2\pi \odot$$

$$\therefore \boxed{\vec{B} = \frac{\mu_0 I}{2R} \odot}$$