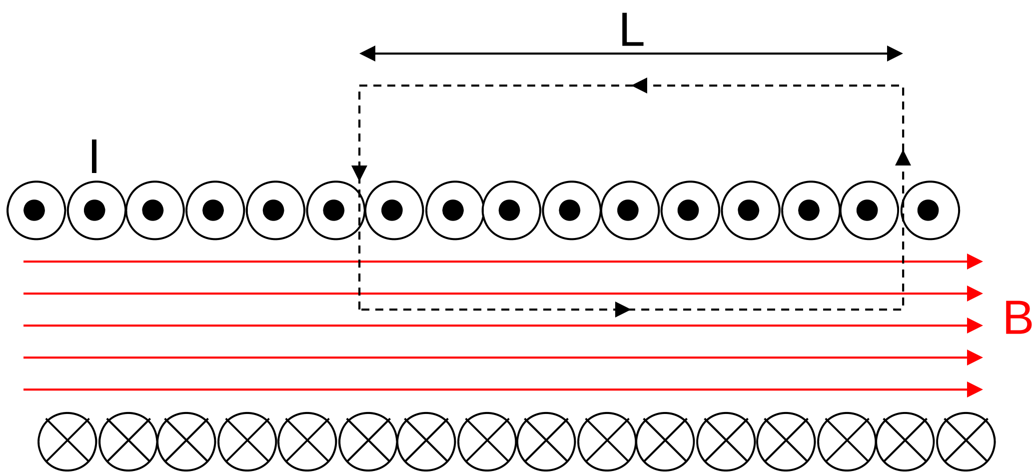
Solenoid

Solenoid



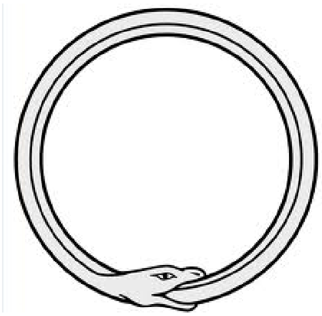
If n = number of turns per unit length

$$\therefore \oint \vec{B} \cdot d\vec{s} = B \cdot L$$

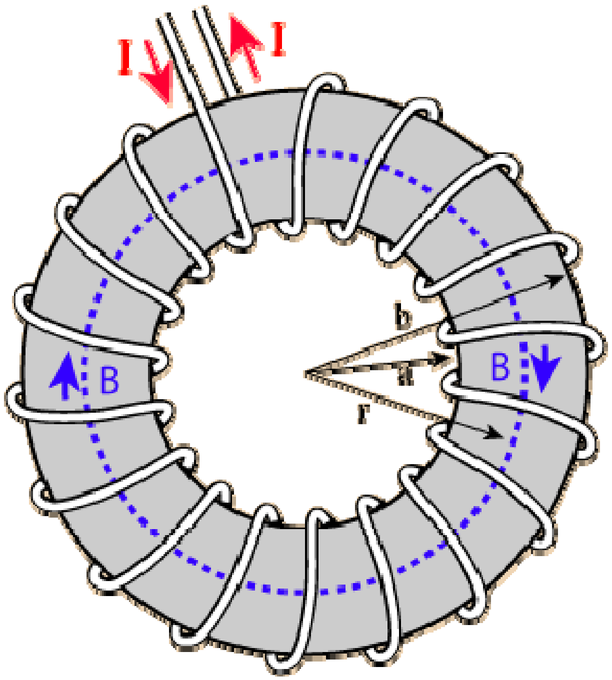
Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 \, I \Rightarrow \; B \cdot L = \mu_0 \, (nL) \, I$

$$\Rightarrow \boxed{B = \mu_0 \, n \, I}$$

Note that B is proportional to the number of turns per unit length, but not the total number of turns.



Toroid



$$\therefore \oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi\, r$$

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0\, I \Rightarrow \quad B \cdot 2\pi\, r = \mu_0\, (n \cdot 2\pi\, r)\, I$$

$$\Rightarrow \; B = \mu_0 \left(\frac{N}{2\pi\, r} \right) I$$

N = Total number of turns

Structure of Equations

E	B
Interaction between charges	Interaction between moving charges/ currents
Coulomb's Law $d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$	Biot-Savart Law $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2}$
Gauss's Law $\varepsilon_0 \oiint \vec{E}\cdot d\vec{A} = q_{in}$	Gauss's Law (Conceptual) $\oiint \vec{B}\cdot d\vec{A} = 0$
Gauss's Law $\varepsilon_0 \oiint \vec{E}\cdot d\vec{A} = q_{in}$	Ampere's Law (Calculation) $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in}$
Parallel capacitor gives uniform E field $\vec{E} = \frac{\sigma}{\varepsilon_0}$	Solenoid gives uniform B field $\vec{B} = \mu_0 nI$

Maxwell's Equations

Maxwell's equations describe all the properties of electric and magnetic fields and there are four equations in it:

	Integral form	Differential form (optional)	Name of equation
1st Equation	$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{enclosed}$	$\varepsilon_0 \nabla \cdot \vec{E} = \rho$	Electric Gauss's Law
	$\oint \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$	Magnetic Gauss's Law
	$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}$	$\nabla \times \vec{B} = \mu_0 \vec{J}$	Ampere's Law (Incomplete)
	Not yet 2nd Equation		

Lorentz force equation is not part of Maxwell's equations. It describes what happens when charges are put in an electric or magnetic fields:

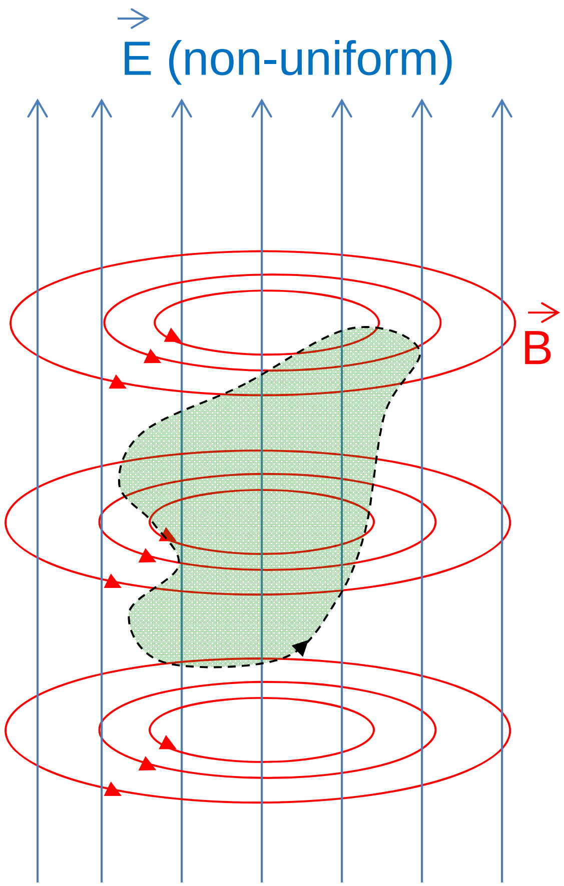
3rd Equation

$$\vec{F} = (q\vec{E} + \vec{v} \times \vec{B})$$

Class 35: Faraday's Law

Part I – Maxwell's 4^{th} Equation

Imaginary loop in an electric and magnetic field



We will do two types of integrals for the closed loop:

1. Magnetic flux

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

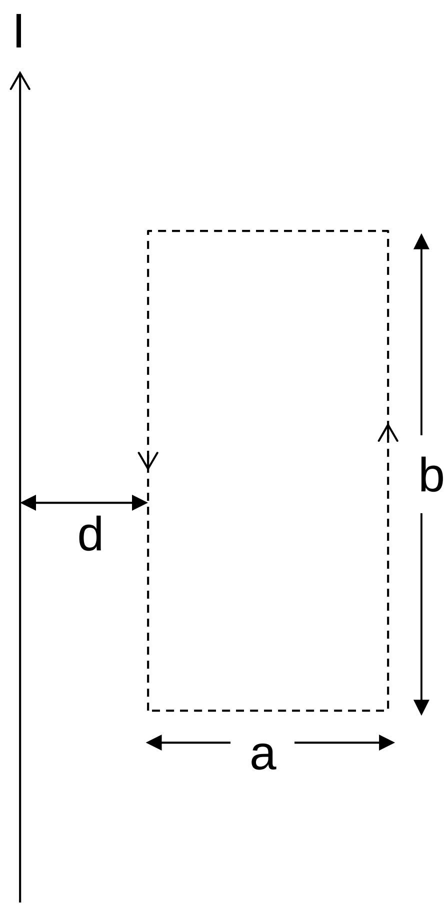
Note that $\Phi_B \neq 0$ (Maxwell's 2nd equation) because this is not a 3 dimensional closed surface.

2. Electromotive force (emf, ε)

$$\varepsilon_{\text{loop}} = \oint_{\text{loop}} \vec{E} \cdot d\vec{s}$$

ε_{loop} = 0 for electrostatic case. Note that ε_{loop} = 0 does not mean E =0.

Example

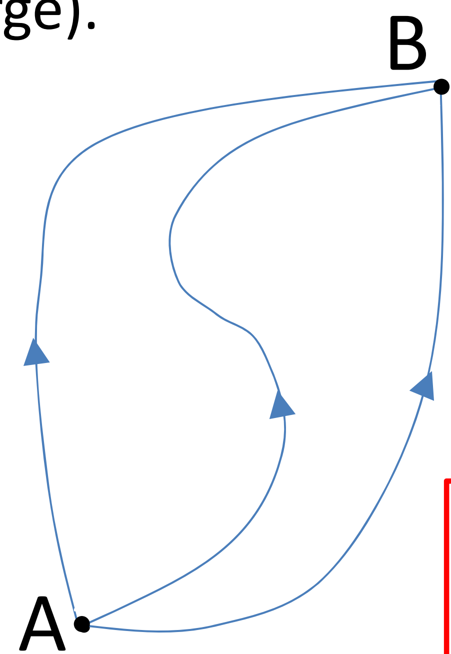


What is the magnetic flux through the rectangular loop?

Old slide from class 13

Electric Potential V

If $\vec{E}(\vec{r})$ is conservative, the potential difference ΔV is defined as the <u>*negative*</u> work done by the force $\vec{F}(\vec{r})$ (which is path independent), divided by the charge (of the test charge).



$$\Delta U = -\int_i^f \vec{F}(\vec{r}) \cdot d\vec{r}$$

Pay attention to the negative sign

$$\Delta V = \frac{\Delta U}{q} = -\int_i^f \vec{E}(\vec{r}) \cdot d\vec{r}$$

$\Delta V=0$ for closed loop

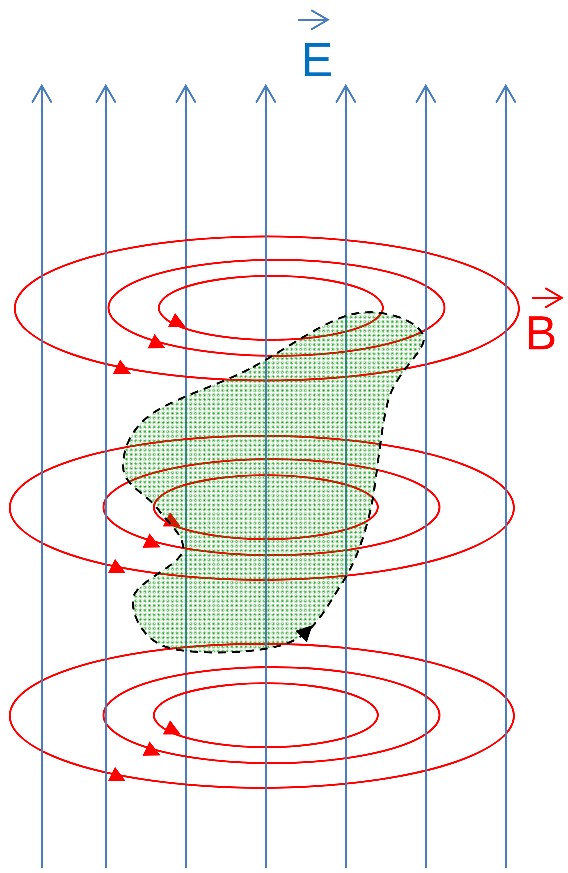
Unit of electric potential = J/C =V

Old slide from class 13

Warning

In the discussion here we will assume electric (force) field is a conservative (force) field. This will not be the case if there is a changing magnetic field. We will come to this point later in the semester.

Faraday's Law – Part 1 (Maxwell's 4th equation)



A changing magnetic field will produce an electric field and they have the following relationship:

$$\varepsilon_{loop} = -\frac{\partial}{\partial t}\Phi_B$$

or

$$\oint_{loop} \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\left(\int \vec{B}\cdot d\vec{A}\right)$$

Notes:

1. We find a new way to produce an electric field.
2. ε_{loop} of electric fields produced this way does not equal to 0.