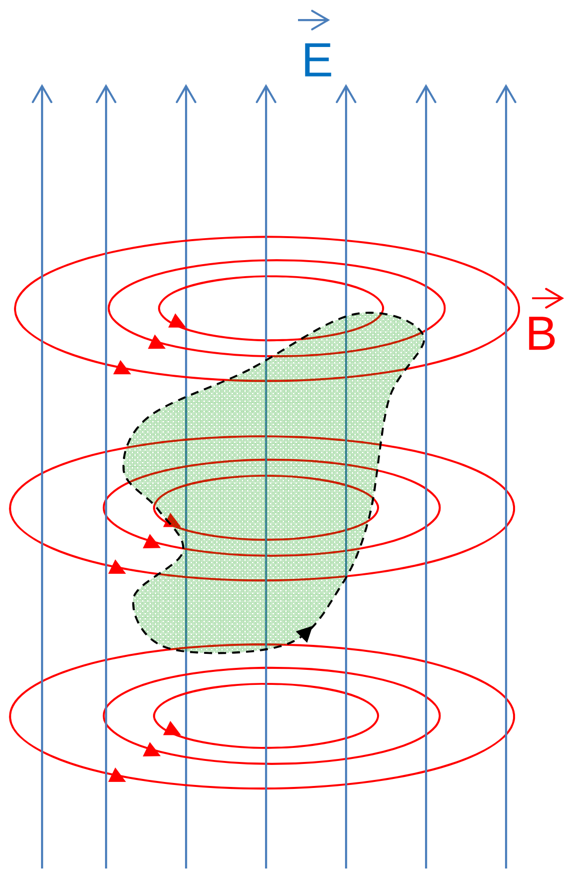
Faraday's Law
Part I – Maxwell's 4th Equation

Faraday's Law – Part 1 (Maxwell's 4th equation)



A changing magnetic field will produce an electric field and they have the following relationship:

$$\varepsilon_{loop} = -\frac{\partial}{\partial t}\Phi_B$$

or

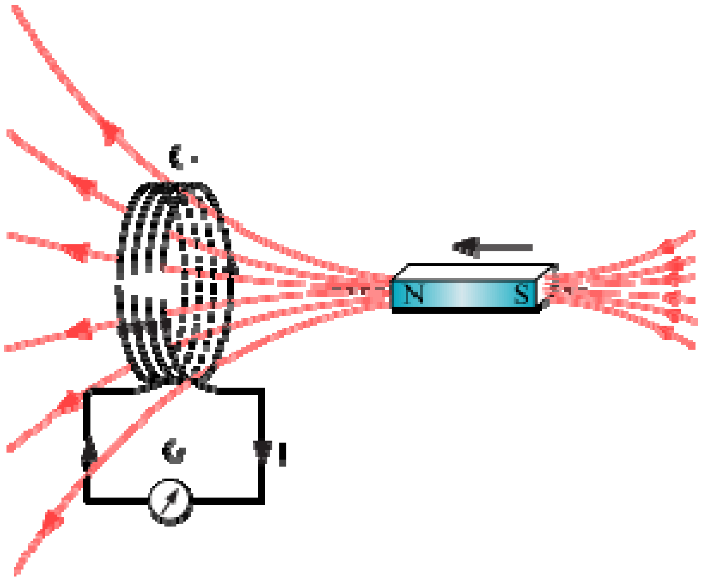
$$\oint_{loop} \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t}\left(\int \vec{B} \cdot d\vec{A}\right)$$

Notes:

1. We find a new way to produce an electric field.
2. ε_{loop} of electric fields produced this way does not equal to 0.

Faraday's Law for changing magnetic field: Example I

$$\varepsilon_{\text{loop}} = -\frac{\partial}{\partial t}\Phi_B$$



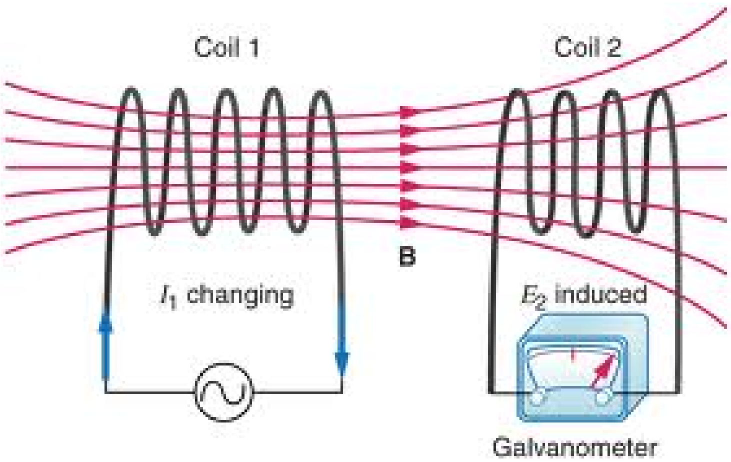
or $$\oint_{\text{loop}} \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\left(\int \vec{B}\cdot d\vec{A}\right)$$

Notes:

1. ε_{loop} does not equal to 0 any more if $\partial\Phi_B/\partial t \neq 0$
2. One way to make $\partial\Phi_B/\partial t \neq 0$ is to change B (i.e. B is a function of time).

Faraday's Law for changing magnetic field: Example 2

$$\varepsilon_{\text{loop}} = -\frac{\partial}{\partial t}\Phi_B$$

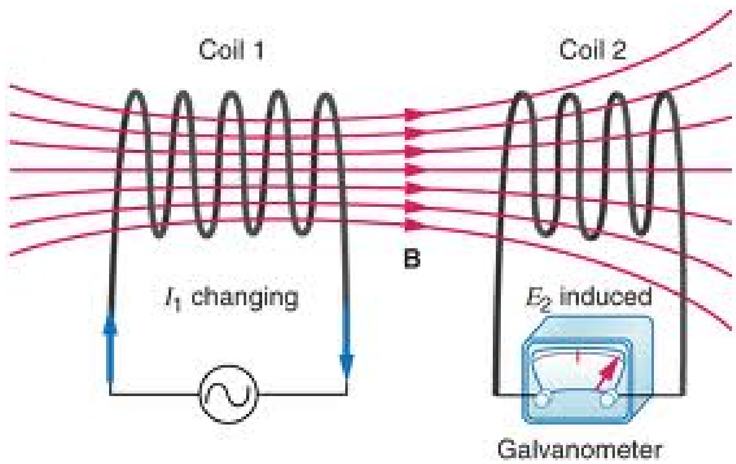


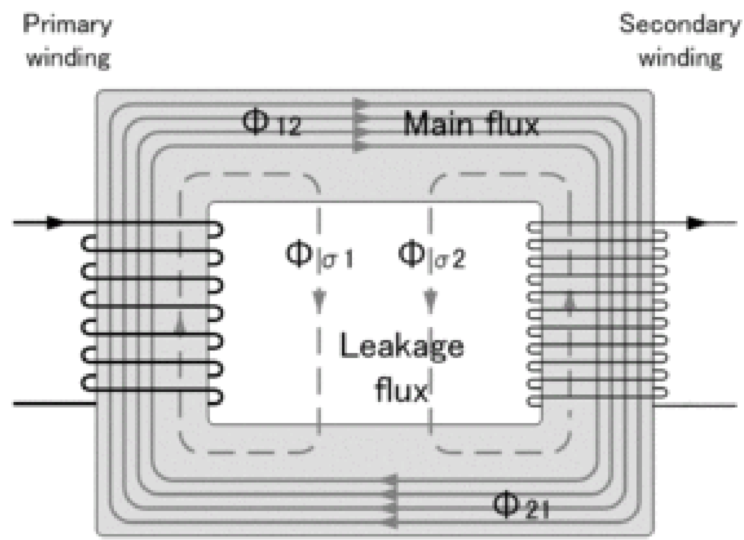
or
$$\oint_{\text{loop}} \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\left(\int \vec{B}\cdot d\vec{A}\right)$$

Notes:

1. ε_{loop} does not equal to 0 any more if $\partial\Phi_B/\partial t \neq 0$
2. One way to make $\partial\Phi_B/\partial t \neq 0$ is to change B (i.e. B is a function of time).

Faraday's Law for changing magnetic field: Transformer





Maxwell's Equations

Maxwell's equations describe all the properties of electric and magnetic fields and there are four equations in it:

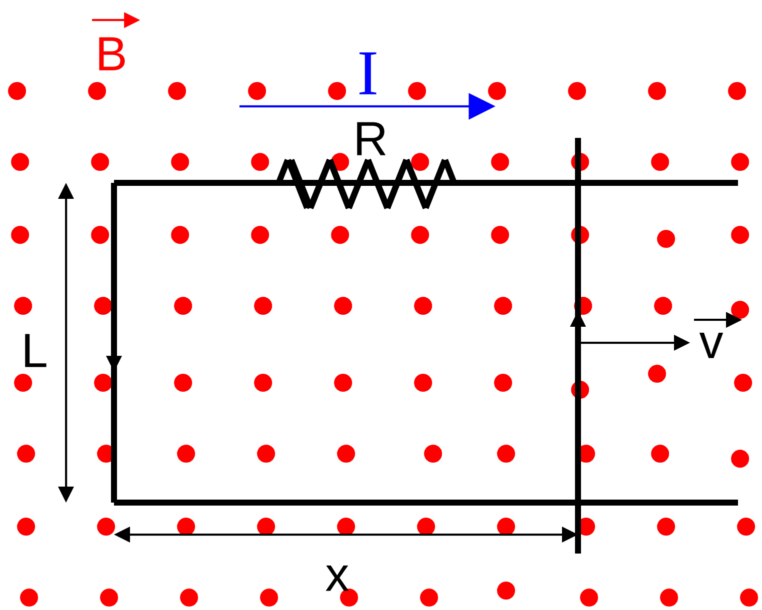
	Integral form	Differential form (optional)	Name of equation
1st Equation	$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{enclosed}$	$\varepsilon_0 \nabla \cdot \vec{E} = \rho$	Electric Gauss's Law
	$\oint \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$	Magnetic Gauss's Law
	$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}$	$\nabla \times \vec{B} = \mu_0 \vec{J}$	Ampere's Law (Incomplete)
	$\underbrace{\oint \vec{E} \cdot d\vec{\ell}}_{\varepsilon} = -\frac{\partial}{\partial t} \underbrace{\oint \vec{B}(t) \cdot d\vec{A}}_{\Phi_B}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	

Lorentz force equation is not part of Maxwell's equations. It describes what happens when charges are put in an electric or magnetic fields:

$$\vec{F} = (q\vec{E} + \vec{v} \times \vec{B})$$

Class 36 Faraday's Law Part II
Motional emf

Faraday's Law for motion emf: A note for Example I



You need an external force to maintain a constant velocity, because of the magnetic field.

You can calculate this force either by

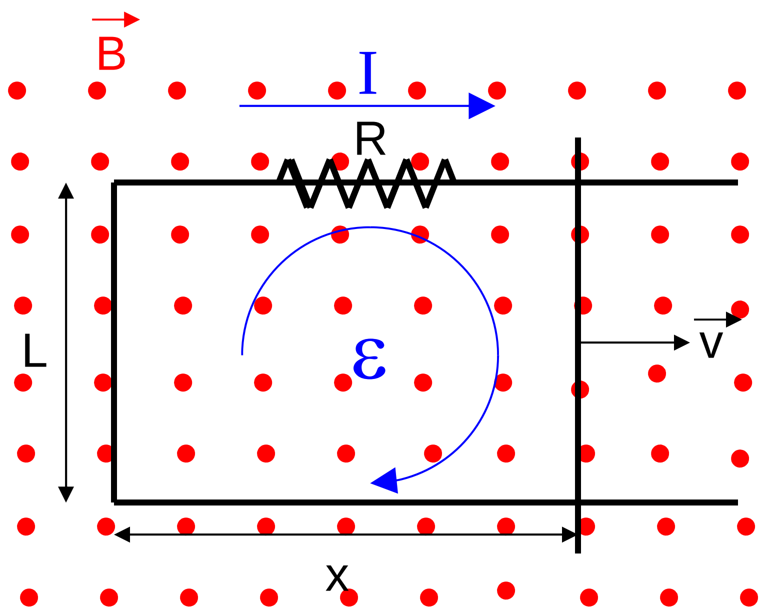
(i) Newton's Law of motion:

$$\vec{F} = - \vec{F}_B$$

(ii) Conservation of energy:

$$I^2R = Fv$$

Faraday's Law for motion emf: Example I – nothing new



Assuming there is an induced emf ε.

$$\therefore F_B = I\vec{L} \times \vec{B} = ILB \quad \leftarrow$$

$$\text{Pulling force} = F_P = -F_B = ILB \rightarrow$$

$$\text{Power of Pulling force} = \vec{F}_P \cdot \vec{v}$$

$$= ILBv$$

But power of Pulling force

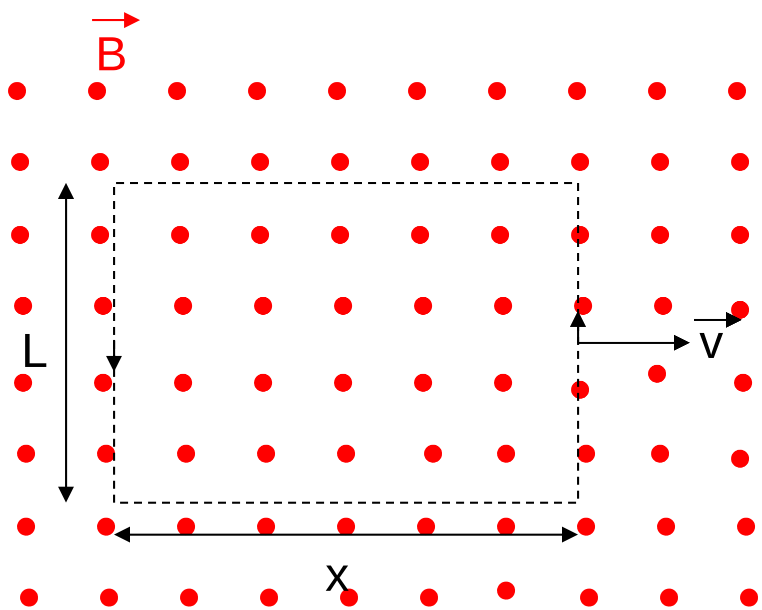
= power dissipated at resistance R

$$ILBv = \frac{|\varepsilon|^2}{R} \quad \Rightarrow |\varepsilon|^2 = \underbrace{RI}_{=|\varepsilon|}LBv$$

$$\Rightarrow |\varepsilon| = LBv$$

Motion emf is just a result of Lorentz force acting on the charge carriers due to the magnetic field.

Faraday's Law for motion emf : Example I – New approach



However, we can rewrite previous result as :

$$|\varepsilon| = LBv$$

$$= BL\left|\frac{dx}{dt}\right|$$

$$= B\left|\frac{d(Lx)}{dt}\right|$$

$$= B\left|\frac{dA}{dt}\right|$$

$$= \left|\frac{d\Phi_B}{dt}\right|$$

Including sign, $\varepsilon = -\dfrac{d\Phi_B}{dt}$

This merges and has the same form as the Faraday's Law for changing B field!

The Faraday's Law

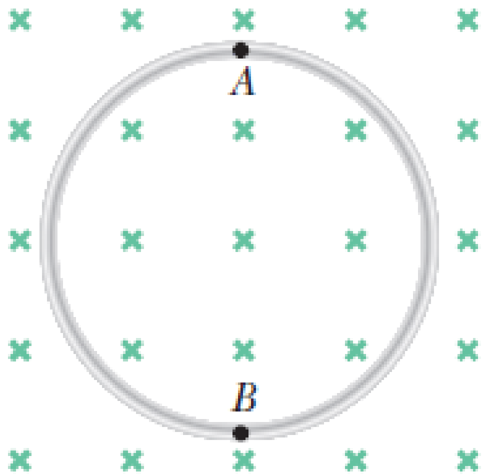
So the two parts of Faraday's Law can be written in one single equation:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Φ_B depends on $\vec{B}$ and $\vec{A}$:

1. If you change $\vec{B}$, you will get the Maxwell's 4th equation.
2. If you change $\vec{A}$, you will get the motion emf.

Faraday's Law for motion emf: Example II



The flexible loop in the figure has a radius and is in a magnetic field of magnitude B. The loop is grasped at points *A* and *B* and stretched until its area is nearly zero. If it takes Δt to close the loop, what is the magnitude of the average induced emf in it during this time interval?