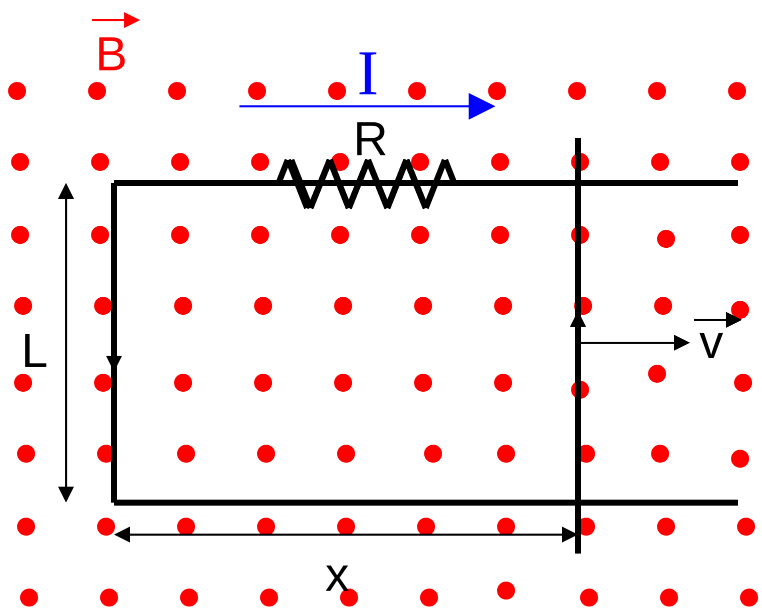
Faraday’s Law Part II - Motional emf

Faraday's Law for motion emf: A note for Example I



You need an external force to maintain a constant velocity, because of the magnetic field.

You can calculate this force either by

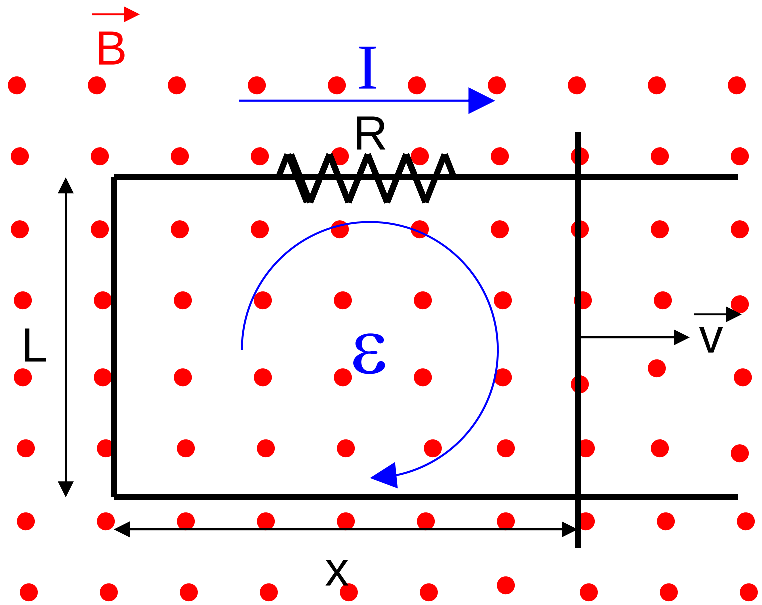
(i) Newton's Law of motion:

$$\vec{F} = -\vec{F}_B$$

(ii) Conservation of energy:

$$I^2R = Fv$$

Faraday's Law for motion emf: Example I – nothing new



Assuming there is an induced emf ε.

$$\therefore F_B = I\vec{L} \times \vec{B} = ILB \quad \leftarrow$$

$$\text{Pulling force} = F_P = -F_B = ILB \rightarrow$$

$$\text{Power of Pulling force} = \vec{F}_P \cdot \vec{v} = ILBv$$

But power of Pulling force

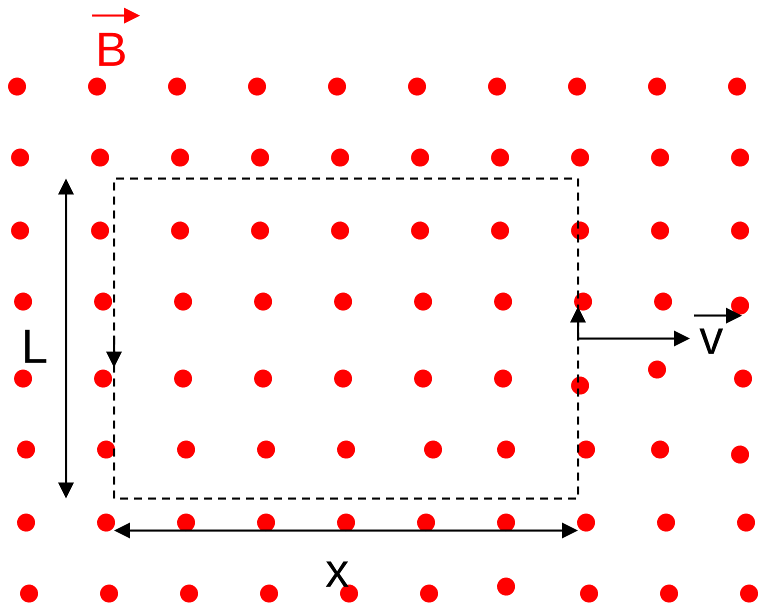
= power dissipated at resistance R

$$ILBv = \frac{|\varepsilon|^2}{R} \Rightarrow |\varepsilon|^2 = \underbrace{RI}_{=|\varepsilon|}LBv$$

$$\Rightarrow |\varepsilon| = LBv$$

Motion emf is just a result of Lorentz force acting on the charge carriers due to the magnetic field.

Faraday's Law for motion emf : Example I – New approach



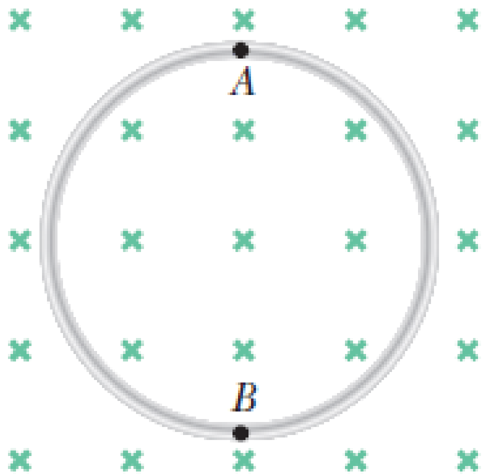
However, we can rewrite previous result as :

$$
\begin{aligned}
|\varepsilon| &= LBv \\
&= BL\left|\frac{dx}{dt}\right| \\
&= B\left|\frac{d(Lx)}{dt}\right| \\
&= B\left|\frac{dA}{dt}\right| \\
&= \left|\frac{d\Phi_B}{dt}\right|
\end{aligned}
$$

Including sign, $\varepsilon = -\dfrac{d\Phi_B}{dt}$

This merges and has the same form as the Faraday's Law for changing B field!

Faraday's Law for motion emf: Example II



The flexible loop in the figure has a radius and is in a magnetic field of magnitude B. The loop is grasped at points *A* and *B* and stretched until its area is nearly zero. If it takes Δt to close the loop, what is the magnitude of the average induced emf in it during this time interval?

The Faraday's Law

So the two parts of Faraday's Law can be written in one single equation:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Φ_B depends on $\vec{B}$ and $\vec{A}$:

1. If you change $\vec{B}$, you will get the Maxwell's 4^{th} equation.
2. If you change $\vec{A}$, you will get the motion emf.

Faraday's Law, Maxwell's 4th Equation, and the Lorentz Force Law

Feynman Lectures on Physics Vol. 2 p.17-2:

…We know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of two *different phenomena*. Usually such a beautiful generalization is found to stem from a single deep underlying principle. Nevertheless, in this case there does not appear to be any such profound implication. We have to understand the "rule" as the combined effects of two quite separate phenomena.

Faraday's Law of Induction

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

You need two of these to "derive" the other.

Maxwell's 4th Equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Lorentz Force Law

$$\frac{\vec{F}}{q} = \vec{v} \times \vec{B}$$

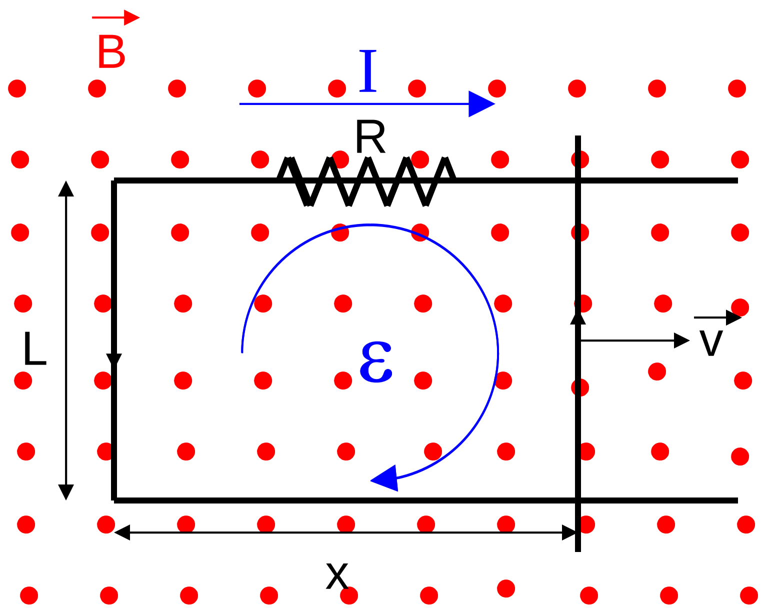
Class 37 Lenz's Law

Lenz's Law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Lenz's Law :
The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.

Lenz’s Law: Example 1



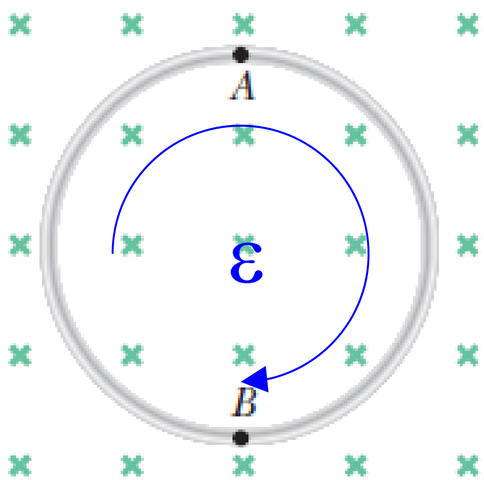
$$\varepsilon_{\text{loop}} = \text{IR}$$

$$\Rightarrow \text{I} = \frac{\varepsilon_{\text{loop}}}{\text{R}} = - \frac{\text{BLv}}{\text{R}}$$

Meaning of negative sign

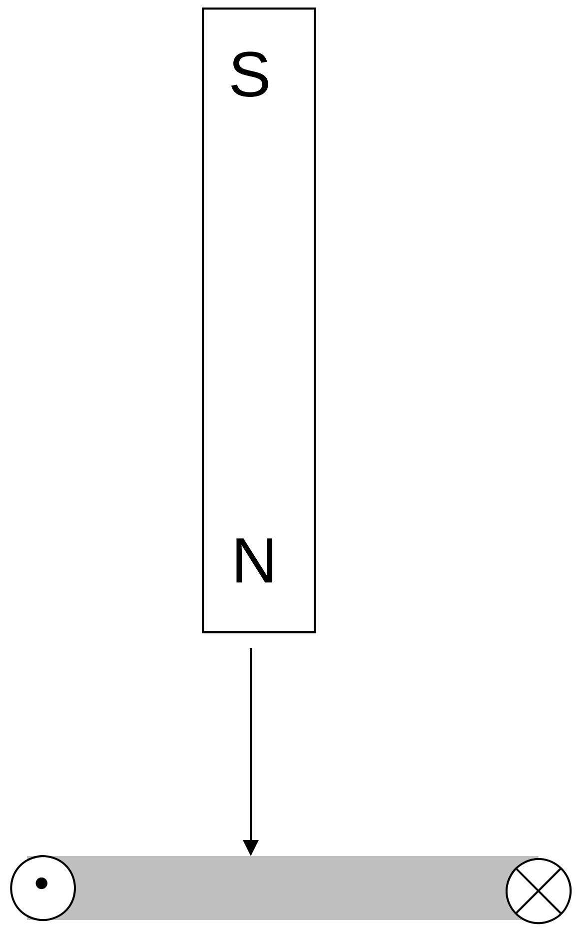
The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.

Lenz's Law: Example 2



The flexible loop in the figure has a radius and is in a magnetic field of magnitude B. The loop is grasped at points *A* and *B* and stretched until its area is nearly zero. If it takes Δt to close the loop, what is the magnitude of the average induced emf in it during this time interval?

Lenz's Law: Example 3



Lenz's Law: Example 4