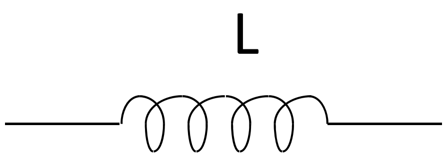
Energy stored in a magnetic field

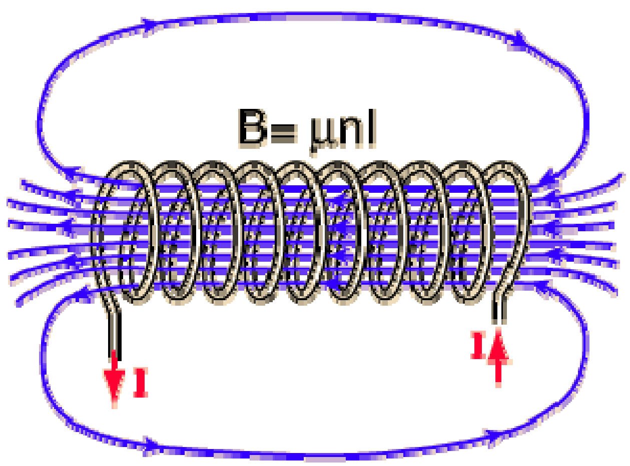
Energy Stored in an Inductor



Energy stored in an inductor:

$$U = \frac{1}{2} L I^2$$

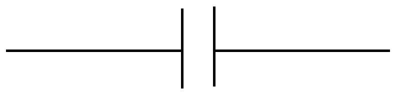
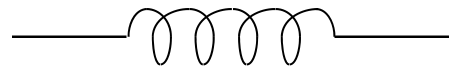
(Do not forget $\varepsilon = - L \frac{dI}{dt}$.)



Energy density stored in an electric field:

$$u_B = \frac{U_B}{\Omega} = \frac{1}{2\mu_0} B^2$$

Capacitor and Inductor

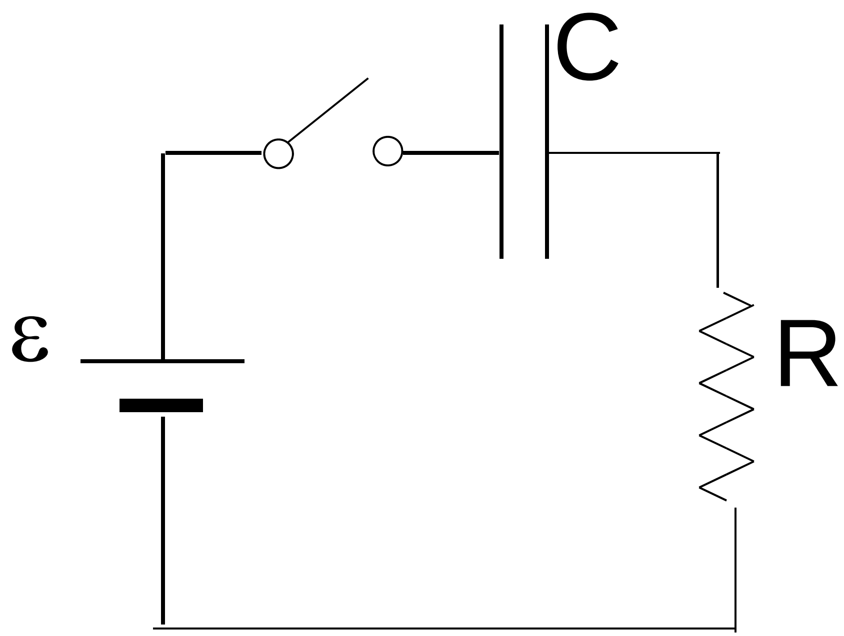
Capacitor C	Inductor L
	
Charge Q	Current I
E field	B field
$V = \frac{Q}{C}$	$\varepsilon = -L\frac{dI}{dt}$
Parallel plate capacitor (uniform E field) $C = \frac{\varepsilon_0 A}{d}$ and $E = \frac{V}{d}$	Solenoid (uniform B field) $L = \mu_0 nNA$ and $B = \mu_0 nI$
$U_E = \frac{1}{2}CV^2$ and $u_E = \frac{1}{2}\varepsilon_0 E^2$	$U_B = \frac{1}{2}LI^2$ and $u_B = \frac{1}{2\mu_0}B^2$

Class 40 RL Circuits

From Class 25

RC Circuits – Charging

Charge



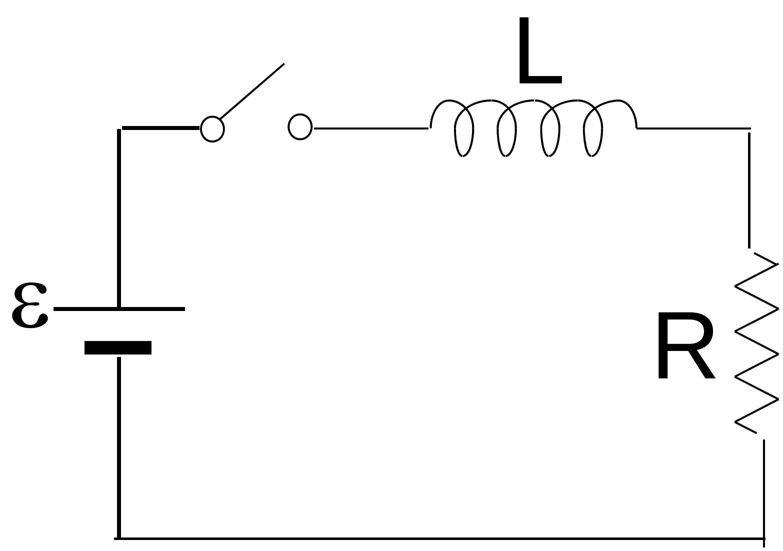
At t=0, capacitance is uncharged and Q=0 (initial condition).

At t=0, switched is closed, if the capacitor has no charge, it behaves like a conductor and $I=\varepsilon/R$.

After the capacitor is completely charged, $Q=C\,\varepsilon$, $\Delta V_C=\varepsilon$ and $\Delta V_R=0$. I=0 and the capacitors behave like an insulator.

RL Circuits – Charging

Current



At t=0, inductor is uncharged and I=0 (initial condition).

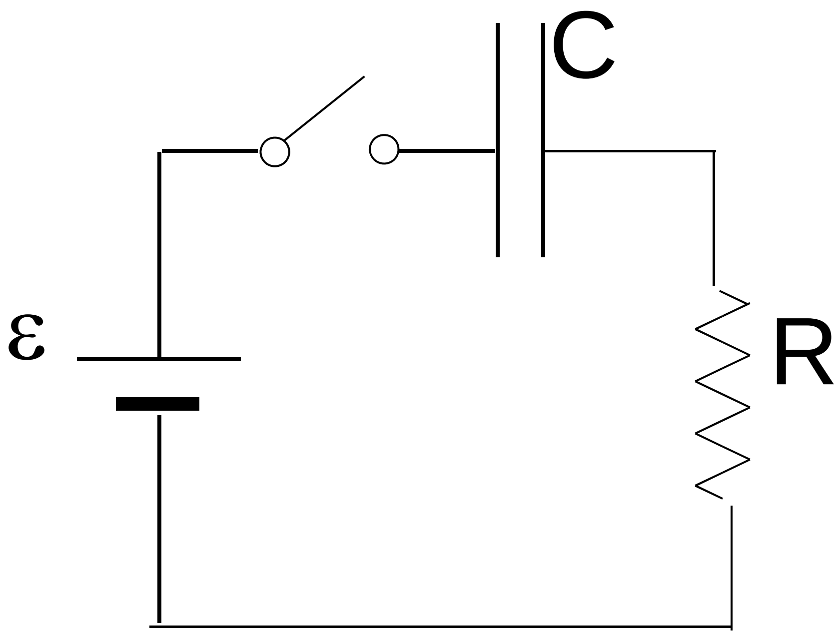
At t=0, switched is closed, if the inductor has no current, it behaves like an insulator (opposes changes) and I=0.

After the inductor is completely charged (with current), $I=\varepsilon/R$, $\Delta V_L = 0$ and $\Delta V_R = \varepsilon$. The inductor behaves like a conductor.

From Class 25

RC Circuits – Charging

← Charge



$$\varepsilon = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = \varepsilon$$

$$\Rightarrow CR\,dq = (C\varepsilon - q)\,dt$$

$$\Rightarrow \frac{dq}{q - C\varepsilon} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n(q - C\varepsilon) = -\frac{t}{CR} + K'$$

$$\Rightarrow q - C\varepsilon = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = C\varepsilon + Ke^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = 0 \Rightarrow 0 = C\varepsilon + K \Rightarrow K = -C\varepsilon$$

$$\therefore q = C\varepsilon(1 - e^{-\frac{t}{CR}})$$

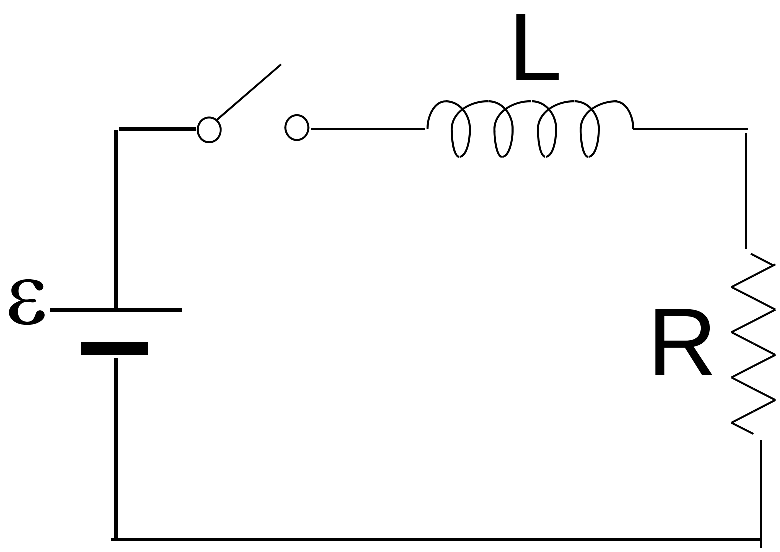
$$I = \frac{dq}{dt} = \frac{C\varepsilon}{CR}e^{-\frac{t}{CR}} = \frac{\varepsilon}{R}e^{-\frac{t}{CR}}$$

$$\Delta V_R = IR = \varepsilon e^{-\frac{t}{CR}}$$

$$\Delta V_C = \frac{q}{C} = \varepsilon(1 - e^{-\frac{t}{CR}})$$

$\Delta V_R + \Delta V_C = \varepsilon$

RL Circuits – Charging ← Current



$$\varepsilon = L\frac{dI}{dt} + IR \Rightarrow L\,dI + I\,R\,dt = \varepsilon\,dt$$

$$\Rightarrow L\,dI = (\varepsilon - IR)dt$$

$$\Rightarrow \frac{L\,dI}{\varepsilon - IR} = dt$$

Integration constant

$$\Rightarrow \frac{L}{R}\ell n(\varepsilon - IR) = t + K'$$

$$\Rightarrow \ell n(\varepsilon - IR) = \frac{R}{L}t + \frac{R}{L}K'$$

$$\Rightarrow \varepsilon - IR = Ke^{-\frac{R}{L}t} \qquad (K = e^{\frac{R}{L}K'})$$

$$\Rightarrow IR = \varepsilon - Ke^{-\frac{R}{L}t}$$

$$\text{At } t = 0, I = 0 \Rightarrow 0 = \varepsilon - K \Rightarrow K = \varepsilon$$

$$\therefore\ I\,R = \varepsilon(1 - e^{-\frac{R}{L}t}) \Rightarrow \underline{\underline{I = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})}}$$

$$\Delta V_R = IR = \underline{\underline{\varepsilon(1 - e^{-\frac{R}{L}t})}}$$

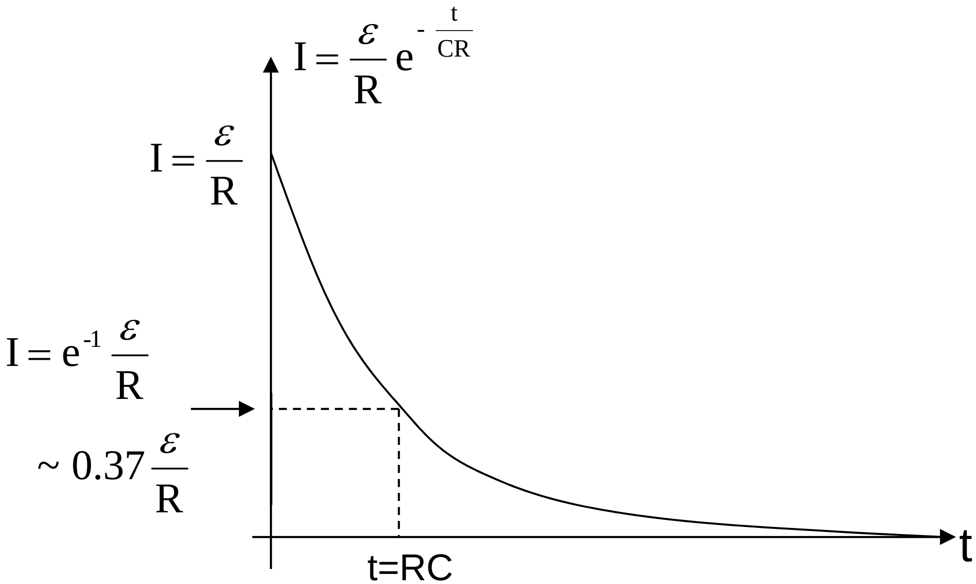
$$\Delta V_L = L\frac{dI}{dt} = \underline{\underline{\varepsilon\,e^{-\frac{R}{L}t}}}$$

$\Delta V_R + \Delta V_C = \varepsilon$

From Class 25

RC time constant

τ=RC is known as the RC time constant. It indicates the response time (how fast you can charge up the capacitor) of the RC circuit.



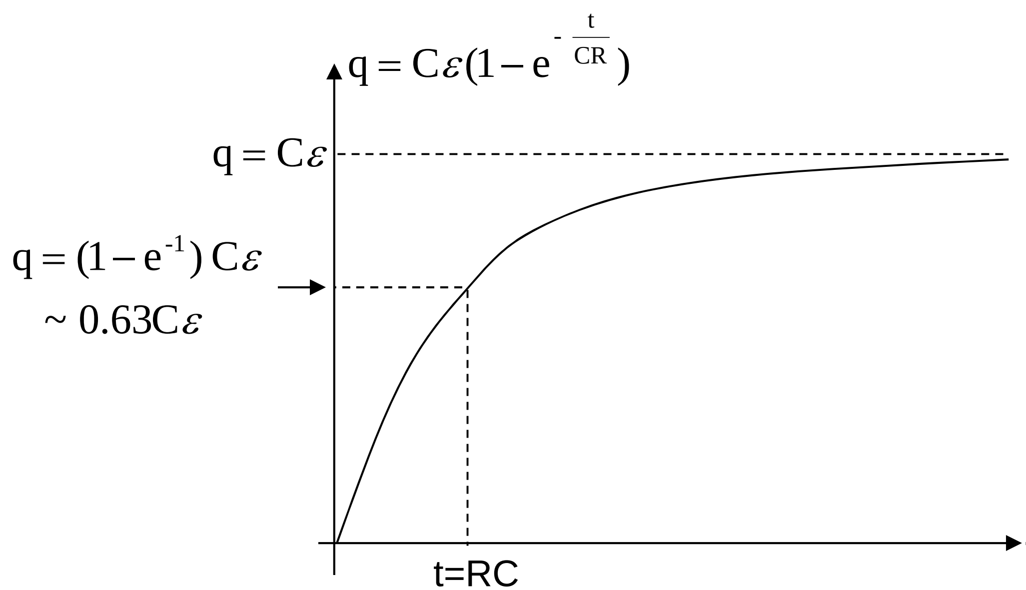
$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$

$$\sqrt{2} \approx 1.414$$

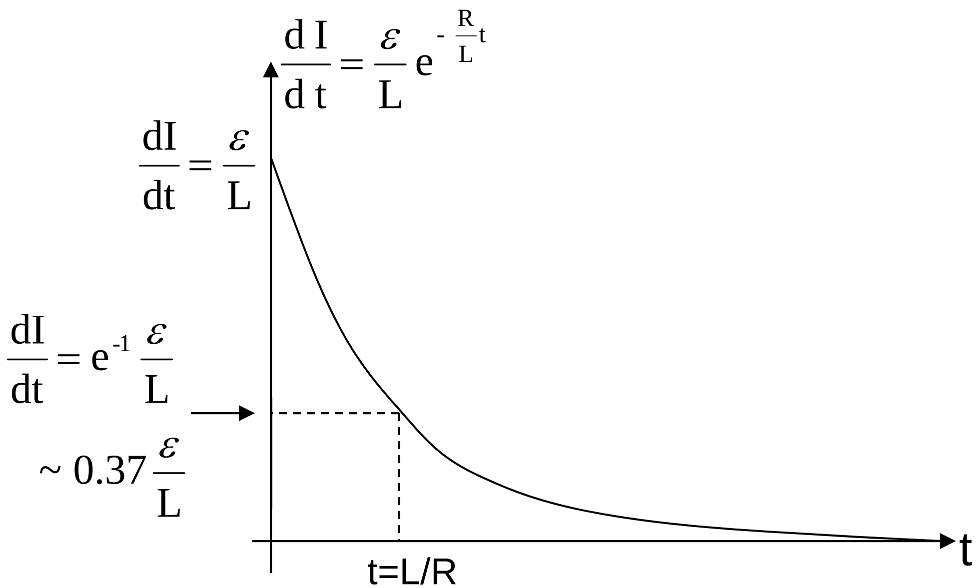
$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RC circuits



L/R time constant

τ=L/R is known as the time constant. It indicates the response time (how fast you can up a current) of the RC circuit.



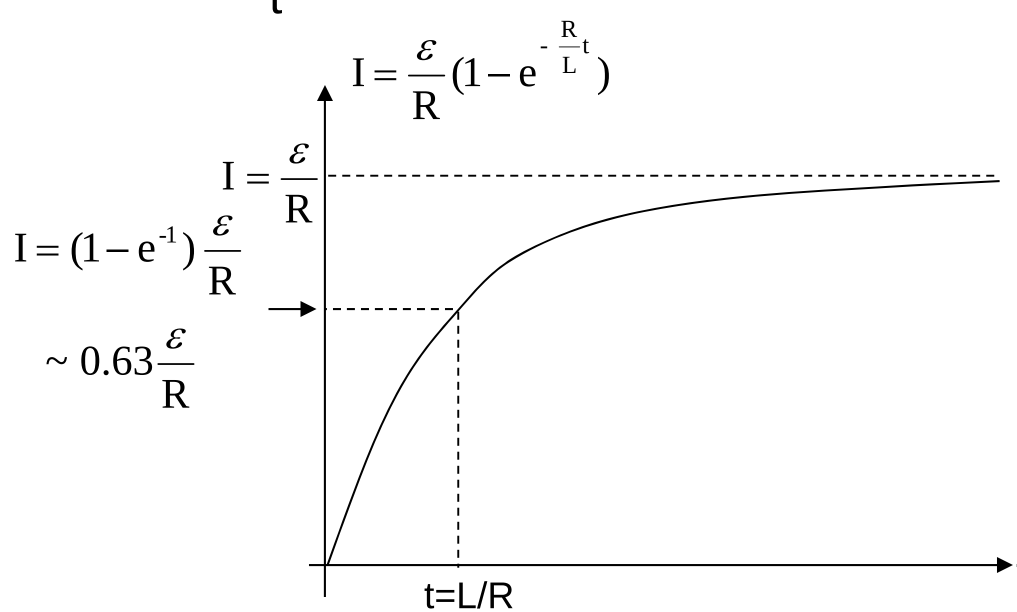
$$e \approx 2.72$$

$$e^{-1} \approx 0.37$$

$$\sqrt{2} \approx 1.414$$

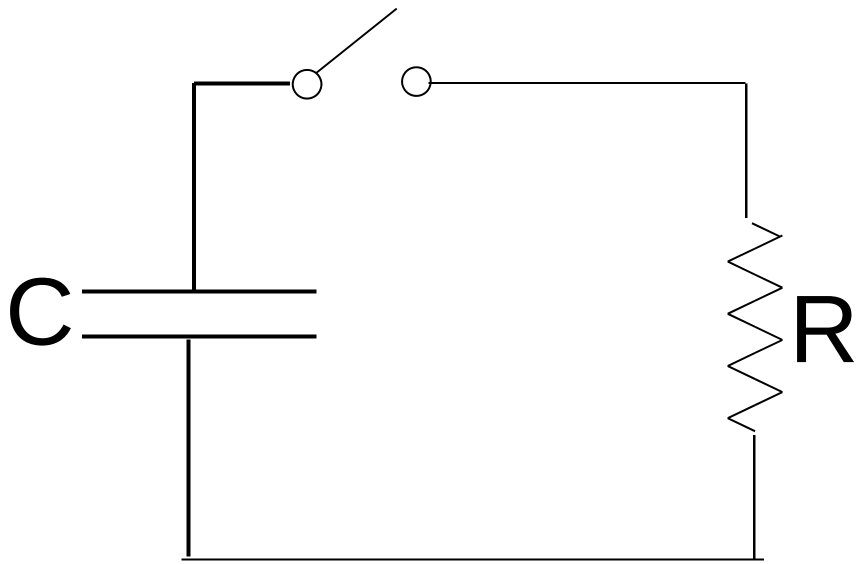
$$\frac{1}{\sqrt{2}} \approx 0.707$$

Nothing to do with RL circuits



From Class 25

RC Circuits – Discharging ← Charge



$$0 = \frac{q}{C} + IR \Rightarrow \frac{q}{C} + R\frac{dq}{dt} = 0$$

$$\Rightarrow CR\,dq = -q\,dt$$

$$\Rightarrow \frac{dq}{q} = -\frac{1}{CR}dt$$

Integration constant

$$\Rightarrow \ell n\, q = -\frac{t}{CR} + K'$$

$$\Rightarrow q = Ke^{-\frac{t}{CR}} \qquad (K = e^{K'})$$

$$\Rightarrow q = K\,e^{-\frac{t}{CR}}$$

$$\text{At } t = 0, q = Q \Rightarrow Q = K$$

$$\therefore\ q = \underline{\underline{Qe^{-\frac{t}{CR}}}}$$

$$I = \frac{dq}{dt} = \underline{\underline{-\frac{Q}{RC}e^{-\frac{t}{CR}}}}$$

$$\Delta V_R = IR = \underline{\underline{-\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$$\Delta V_C = \frac{q}{C} = \underline{\underline{\frac{Q}{C}e^{-\frac{t}{CR}}}}$$

$\Delta V_R + \Delta V_C = 0$