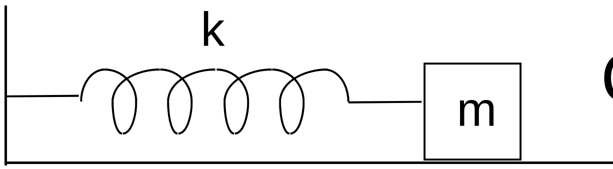
Please Do Course Evaluation

LC and RLC Circuits

Oscillation - Spring

Potential energy $\leftrightarrow$ Kinetic energy

$$\frac{1}{2}kx^2 \qquad \frac{1}{2}mv^2$$



Conservation of energy:

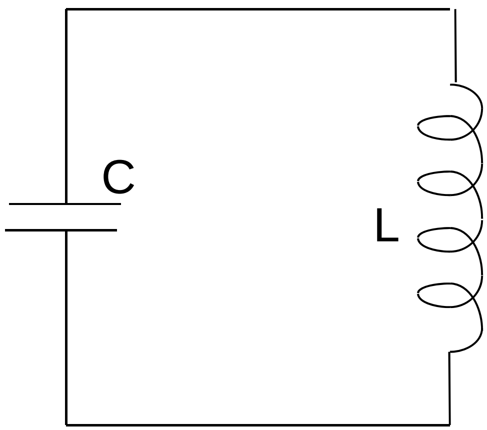
$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{constant}$$

$$= \frac{1}{2}kA^2 \text{ or } \frac{1}{2}mv_{max}^2$$

Equation of motion: $$m\frac{d^2}{dt^2}x = -kx$$

Solution: $$x = A\sin(\omega t + \phi) \text{ with } \omega = \sqrt{\frac{k}{m}}$$

Oscillation – LC circuit



Electric energy $\leftrightarrow$ Magnetic energy

$$\frac{1}{2}\frac{Q^2}{C} \qquad\qquad \frac{1}{2}LI^2$$

Conservation of energy:

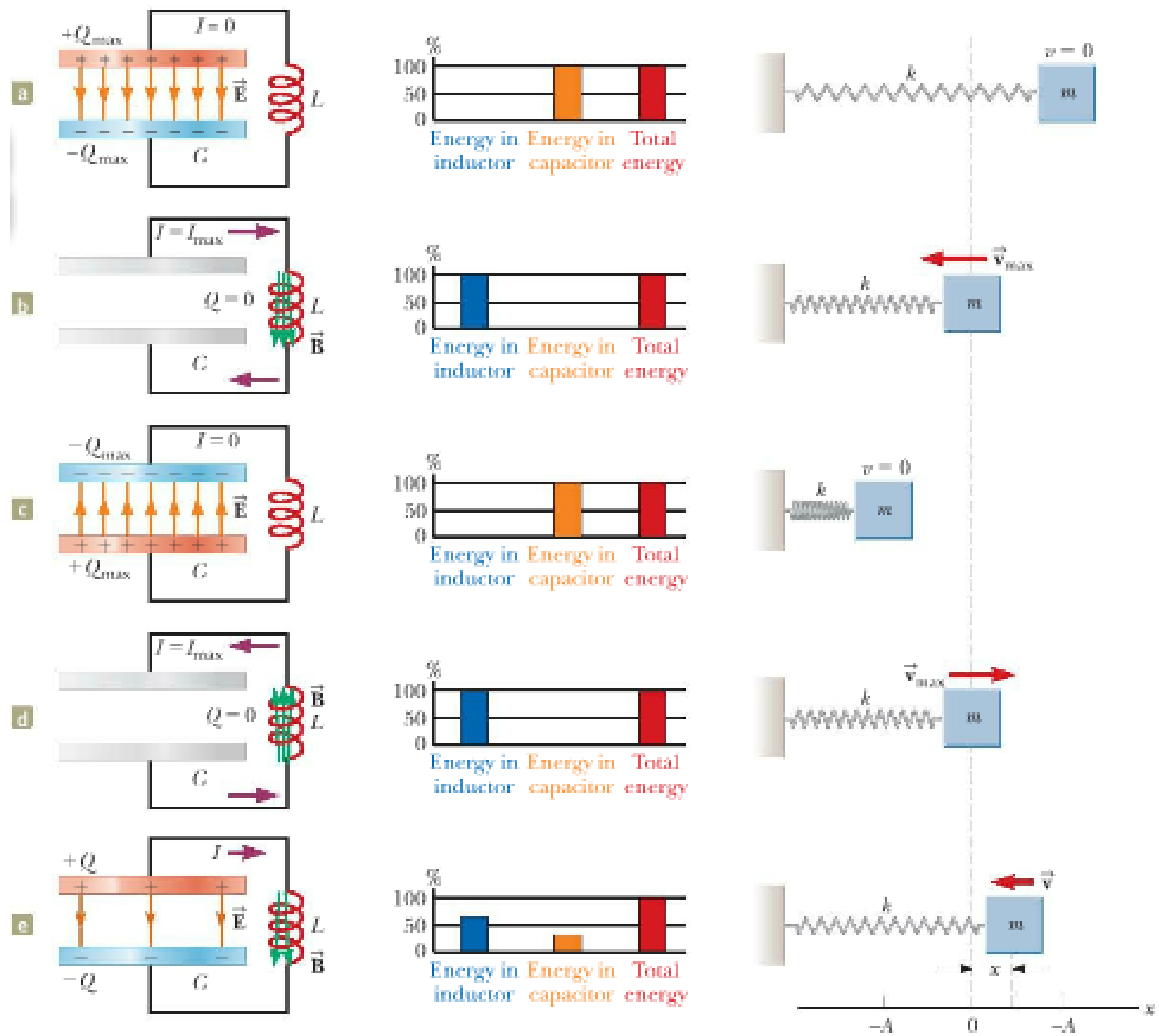
$$\frac{1}{2}\frac{1}{C}Q^2 + \frac{1}{2}LI^2 = \text{constant}$$

$$= \frac{1}{2}\frac{1}{C}{Q_{max}}^2 \text{ or } \frac{1}{2}L{I_{max}}^2$$

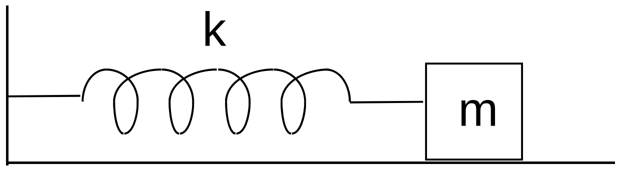
Kirchhoff's rule : $-L\frac{dI}{dt} = \frac{1}{C}Q \Rightarrow \frac{d^2Q}{dt^2} = -\frac{1}{C}Q$

Solution: Solve the differential equation!

Similarity between Spring Oscillation and LC Oscillation I



Similarity between Spring Oscillation and LC Oscillation II

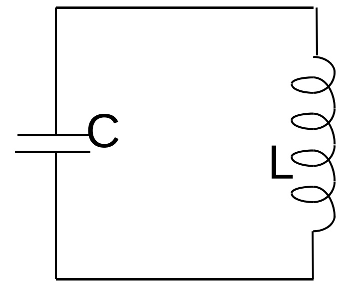


Potential energy $\leftrightarrow$ Kinetic energy

$$\frac{1}{2}kx^2 \qquad \frac{1}{2}mv^2$$

Newton's Law

$$m\frac{d^2}{dt^2}x = -kx$$



Electric energy $\leftrightarrow$ Magnetic energy

$$\frac{1}{2}\frac{Q^2}{C} \qquad \frac{1}{2}LI^2$$

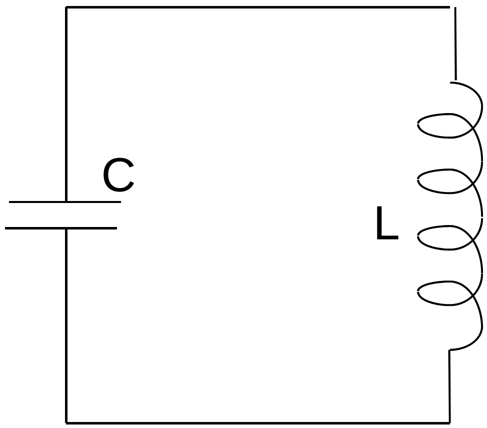
Kirchhoff's rule:

$$-L\frac{dI}{dt} = \frac{1}{C}Q \Rightarrow \frac{d^2Q}{dt^2} = -\frac{1}{C}Q$$

Potential energy	$\frac{1}{2}kx^2$	Electrical energy	$\frac{1}{2}\frac{Q^2}{C}$
Kinetic energy	$\frac{1}{2}mv^2$	Magnetic energy	$\frac{1}{2}LI^2$
Spring constant k		1/Capacitance	$\frac{1}{C}$
Mass m		Inductance L	
Displacement x		Charge Q	
Velocity v	$v = \frac{dx}{dt}$	Current I	$I = \frac{dQ}{dt}$

Class 42 More LC Circuiturrrent

Oscillation – LC circuit



Electric energy $\leftrightarrow$ Magnetic energy

$$\frac{1}{2}\frac{Q^2}{C} \qquad \frac{1}{2}LI^2$$

Conservation of energy:

$$\frac{1}{2}\frac{1}{C}Q^2 + \frac{1}{2}LI^2 = \text{constant}$$

Kirchhoff's rule : $-L\frac{dI}{dt} = \frac{1}{C}Q \Rightarrow L\frac{d^2Q}{dt^2} = -\frac{1}{C}Q$

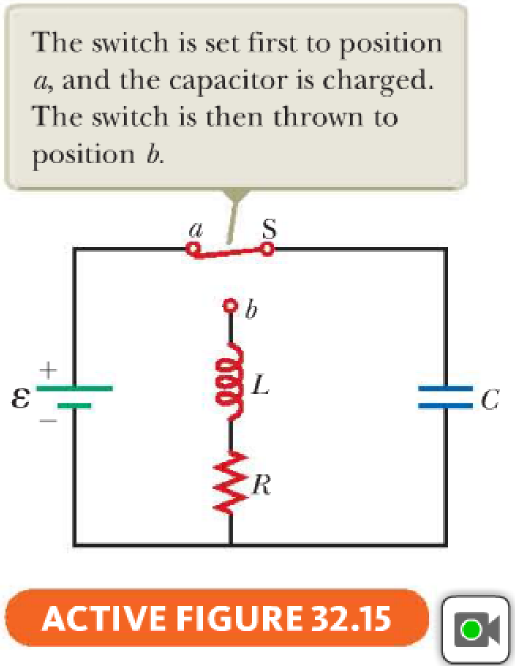
Solution: $x = A\sin(\omega t + \phi)$ with $\omega = \sqrt{\frac{k}{m}}$

$\downarrow$

$$Q = A\sin(\omega t + \phi) \text{ with } \boxed{\omega = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}}$$

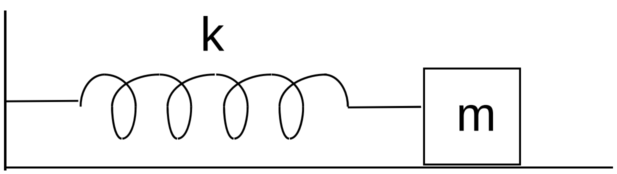
RLC circuit

RLC circuit



A series *RLC* circuit.

Damped Oscillation



Friction = -bv

Kirchhoff's rule :

$$0 = \frac{Q}{C} + IR + L\frac{dI}{dt} \qquad (I = \frac{d}{dt}Q)$$

$$\Rightarrow L\frac{d^2}{dt^2}Q + R\frac{d}{dt}Q + \frac{Q}{C} = 0$$

Equation of motion :

$$m\frac{d^2}{dt^2}x = -bv - kx \qquad (v = \frac{d}{dt}x)$$

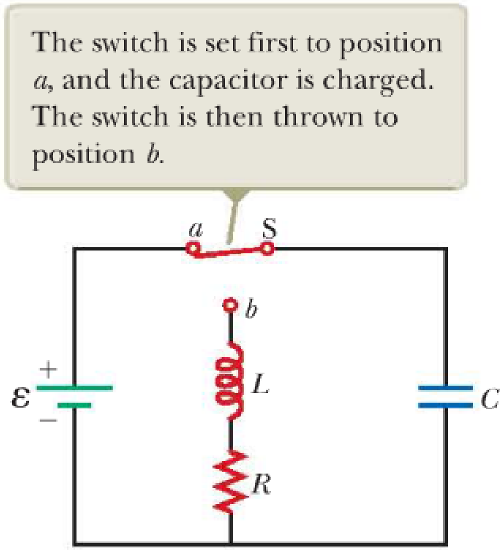
$$\Rightarrow m\frac{d^2}{dt^2}x + b\frac{d}{dt}x + kx = 0$$

RLC circuit and Mechanical Oscillation

RLC circuit	Mechanical
Q	x
$I = dQ/dt$	$v = dx/dt$
C	$1/k$
R	b
L	m
Magnetic energy $\frac{1}{2}LI^2$	Kinetic energy $\frac{1}{2}mv^2$
Electrical energy $\frac{1}{2}(1/C)Q^2$	Potential energy $\frac{1}{2}kx^2$

RLC circuit

RLC circuit



ACTIVE FIGURE 32.15

A series *RLC* circuit.

Kirchhoff's rule :

$$0 = \frac{Q}{C} + IR + L\frac{dI}{d\,t} \qquad (I = \frac{d}{dt}Q\,)$$

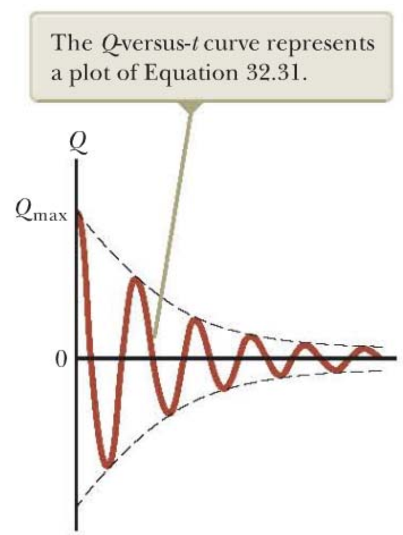
$$\Rightarrow \quad L\frac{d^2}{dt^2}Q + R\frac{d}{dt}Q + \frac{Q}{C} = 0$$

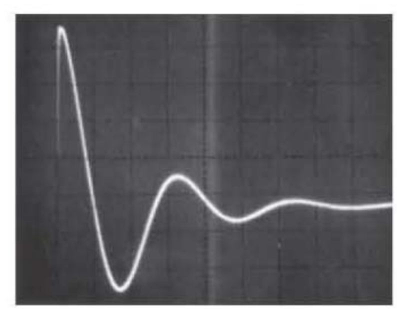
Solution:

$$Q(t) = Q_0\ e^{-\frac{R}{2L}t} \cos \omega_d$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 \begin{cases} > 0 & \text{under damped} \\ = 0 & \text{critically damped} \\ < 0 & \text{over damped} \end{cases}$$





Damping

$$Q(t) = Q_0 \, e^{-\frac{R}{2L}t} \cos \omega_d$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

ω_d real: under damped

$\omega_d = 0$: critically damped

ω_d imaginary: overdamped