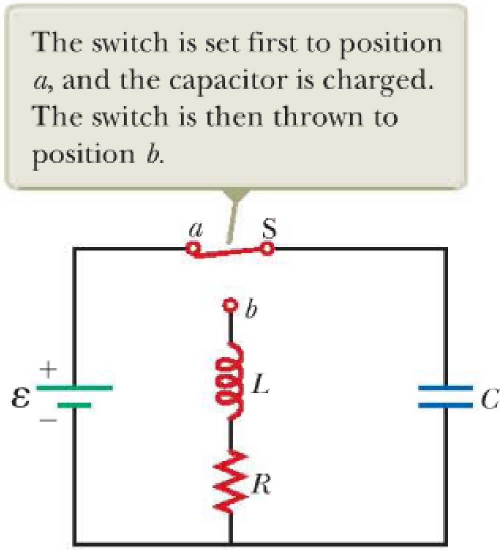
Please Do Course Evaluation

RLC circuit

RLC circuit



ACTIVE FIGURE 32.15

A series *RLC* circuit.

Kirchhoff's rule :

$$0 = \frac{Q}{C} + IR + L\frac{dI}{dt} \qquad (I = \frac{d}{dt}Q)$$

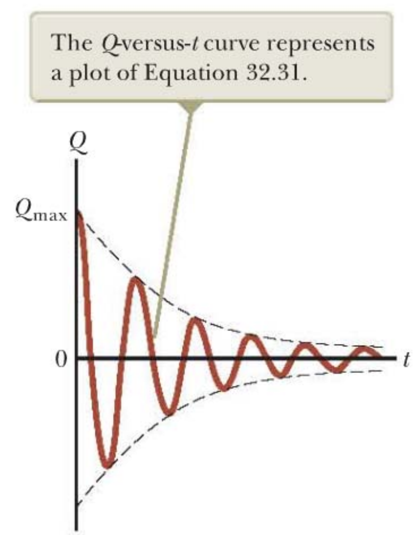
$$\Rightarrow \quad L\frac{d^2}{dt^2}Q + R\frac{d}{dt}Q + \frac{Q}{C} = 0$$

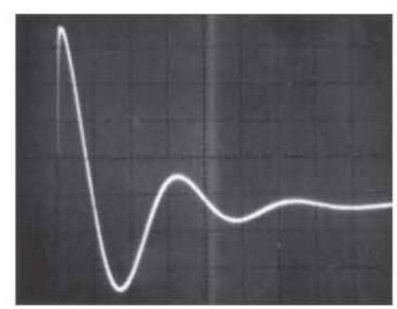
Solution:

$$Q(t) = Q_0\, e^{-\frac{R}{2L}t} \cos \omega_d$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\frac{1}{LC} - \left(\frac{R}{2L}\right)^2 \begin{cases} > 0 & \text{under damped} \\ = 0 & \text{critically damped} \\ < 0 & \text{over damped} \end{cases}$$





Damping

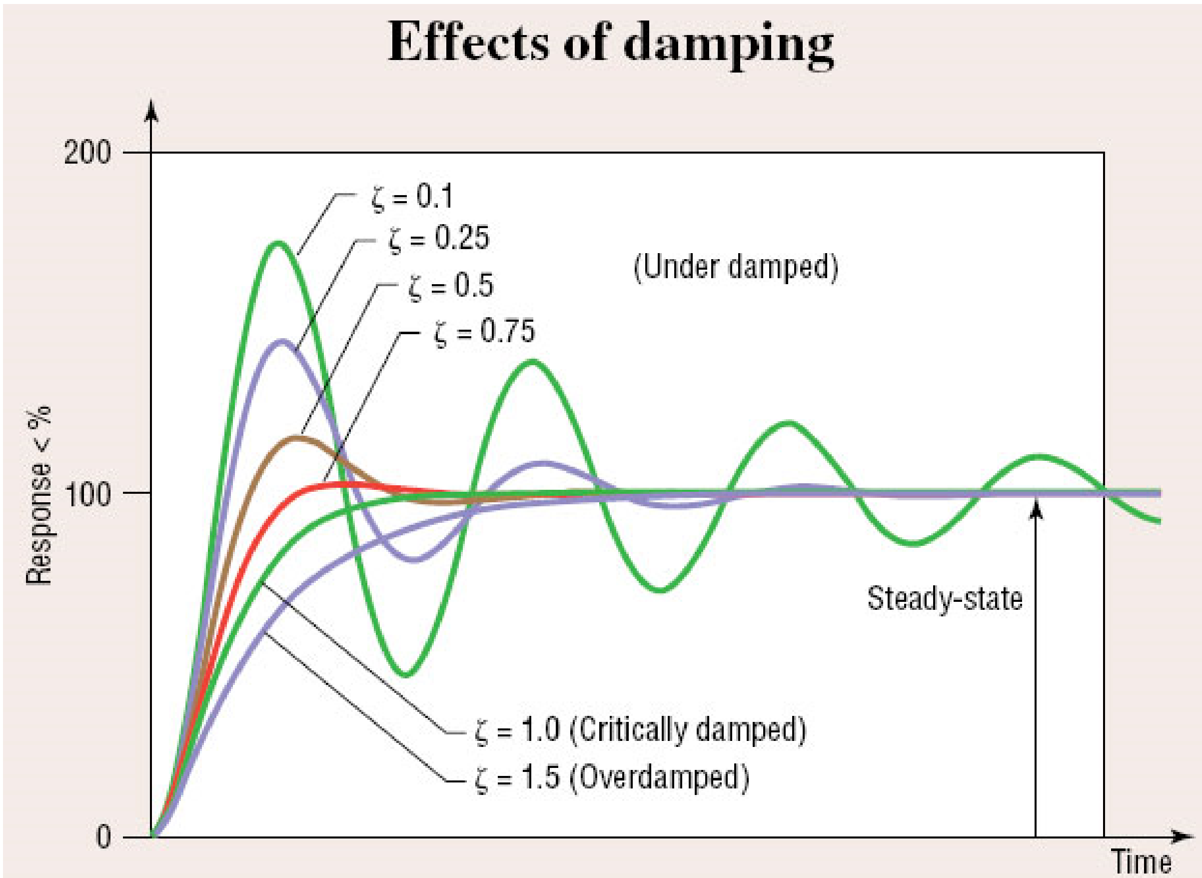
$$Q(t) = Q_0\, e^{-\frac{R}{2L}t} \cos \omega_d$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

ω_d real: under damped

$\omega_d = 0$: critically damped

ω_d imaginary: overdamped



Class 43 Displacement currrent

Maxwell's Equations

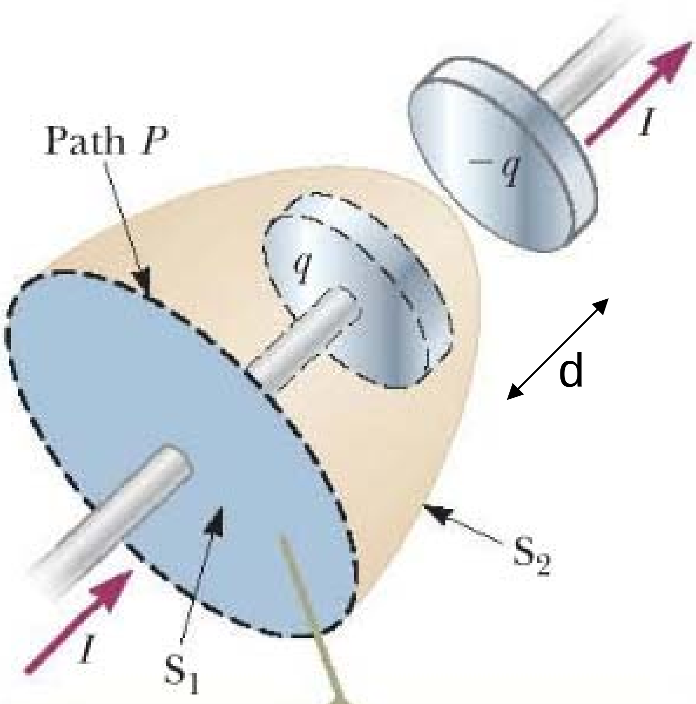
Maxwell's equations describe all the properties of electric and magnetic fields and there are four equations in it:

	Integral form	Differential form (optional)	Name of equation
1st Equation	$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = Q_{enclosed}$	$\varepsilon_0 \nabla \cdot \vec{E} = \rho$	Electric Gauss's Law
	$\oint \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$	Magnetic Gauss's Law
	$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}$	$\nabla \times \vec{B} = \mu_0 \vec{J}$	Ampere's Law (Incomplete)
	$\underbrace{\oint \vec{E} \cdot d\vec{\ell}}_{\varepsilon} = -\frac{\partial}{\partial t} \underbrace{\oint \vec{B}(t) \cdot d\vec{A}}_{\Phi_B}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	

Lorentz force equation is not part of Maxwell's equations. It describes what happens when charges are put in an electric or magnetic fields:

$$\vec{F} = (q\vec{E} + \vec{v} \times \vec{B})$$

Revisit Ampere's Law



The conduction current I in the wire passes only through S_1, which leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S_2.

For DC, I=0 and B=0, so there is no problem.

If I is changing with time, I ≠0 (except at the gap) and there will be a magnetic field (changing with time also).

If the gap d is very small (d →0), there should be magnetic field everywhere surrounding the wire even though there is no physical current through the gap.

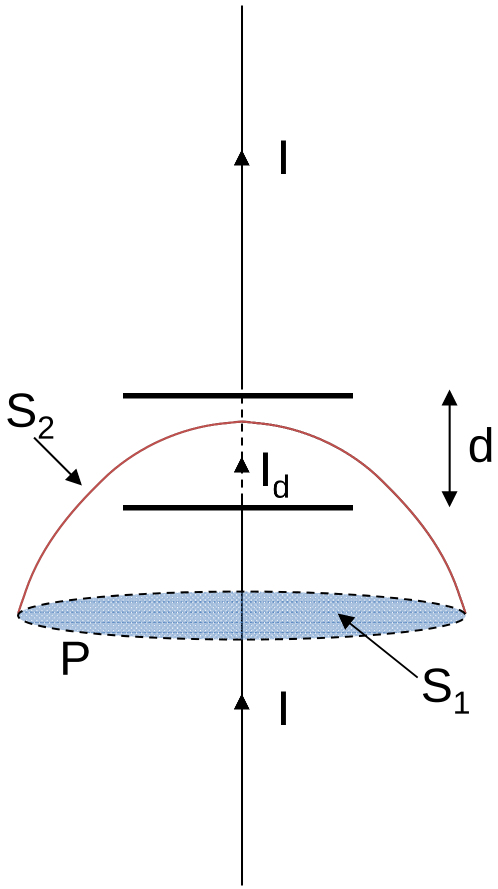
The problem now is:

$$\text{For surface } S_1: \oint_{\text{Path P}} \vec{B}\cdot d\vec{s} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_1}} = \mu_0 I$$

$$\text{For surface } S_2: \oint_{\text{Path P}} \vec{B}\cdot d\vec{s} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_2}} = 0$$

How to reconcile the difference?

Maxwell's proposal



We can introduce an imaginary current, called displacement current, I_d within the gap so the current now looks like continuous.

With this displacement current:

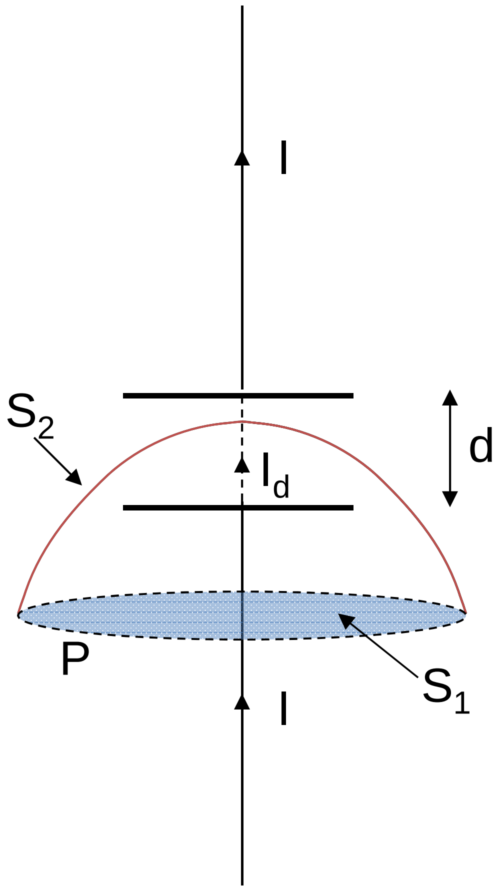
$$\text{For surface } S_1 : \oint_P \vec{B} \cdot d\vec{S} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_1}} = \mu_0 I$$

$$\text{For surface } S_2 : \oint_P \vec{B} \cdot d\vec{S} = \mu_0 I_{\substack{\text{Enclosed} \\ \text{by } S_2}} = \mu_0 I_d = \mu_0 I$$

Ampere's Law now becomes:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 (I_{\text{Enclosed}_1} + I_d)$$

Displacement current



But at the end what is a displacement current?

It is not a real current due to motion of charges within the gap, so we have to relate it to something that really exists in the gap: electric field.

$$I_d = I = \frac{dq}{dt} = C\frac{dV}{dt} \qquad (q = CV)$$

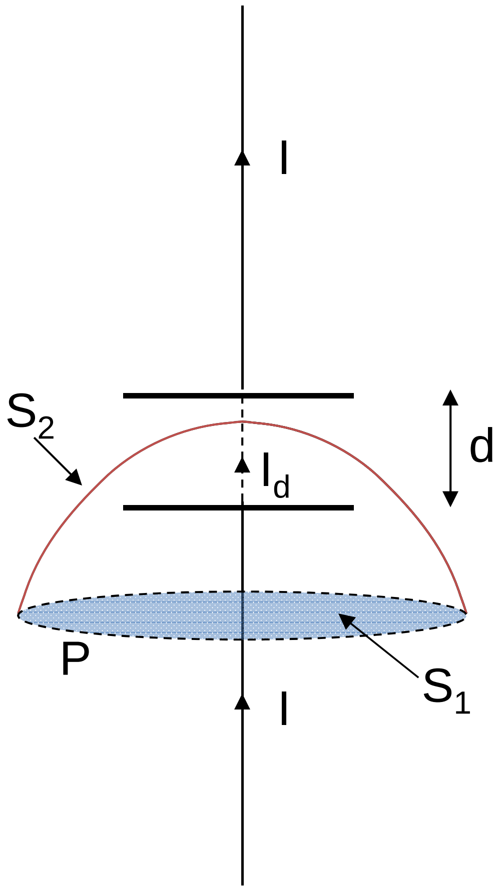
$$= Cd\frac{dE}{dt} \qquad (V = Ed)$$

$$= \frac{\varepsilon_0 A}{d} \cdot d\frac{dE}{dt} \qquad (C = \frac{\varepsilon_0 A}{d})$$

$$= \varepsilon_0 \frac{d(EA)}{dt}$$

$$= \varepsilon_0 \frac{d\Phi_E}{dt}$$

Abstraction



$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

We got this idea from parallel plate capacitor. We expand this and say this is generally true for any geometry and Ampere's Law now becomes:

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 (I_{Enclosed} + \varepsilon_0 \frac{d\Phi_E}{dt})$$

$$= \mu_0 (I_{Enclosed} + \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A})$$

Maxwell's Equations

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	$\underbrace{\oint \vec{E} \cdot d\vec{\ell}}_{\varepsilon} = - \frac{\partial}{\partial t} \underbrace{\oint \vec{B}(t) \cdot d\vec{A}}_{\Phi_B}$	$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$	

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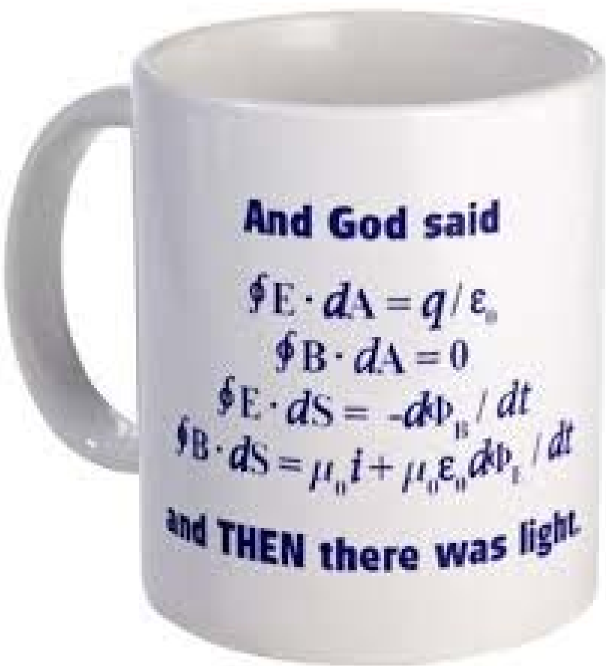
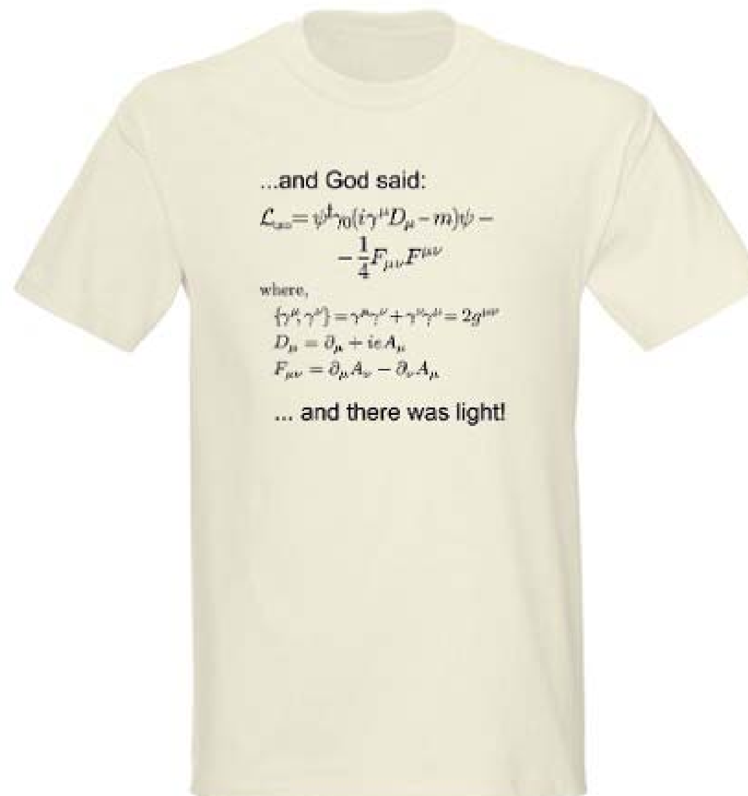
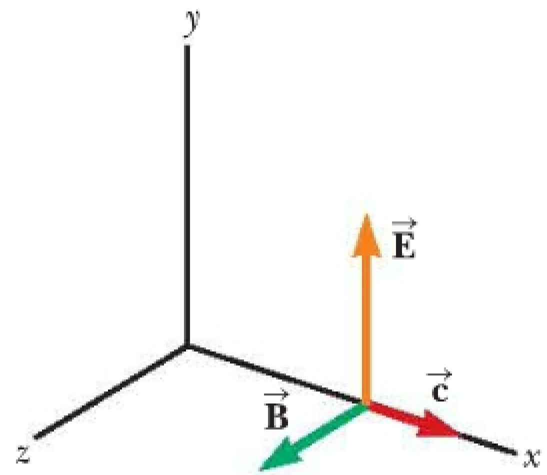
Three different forms of Maxwell’s Equations





Linearly polarized electromagnetic Waves



ACTIVE FIGURE 34.5

Electric and magnetic fields of an electromagnetic wave traveling at velocity $\vec{c}$ in the positive x direction. The field vectors are shown at one instant of time and at one position in space. These fields depend on x and t.

Linearly polarized waves

The wave is traveling in the $\vec{E} \times \vec{B}$ direction.

Applying Maxwell's Third Equation to Plane Electromagnetic Waves

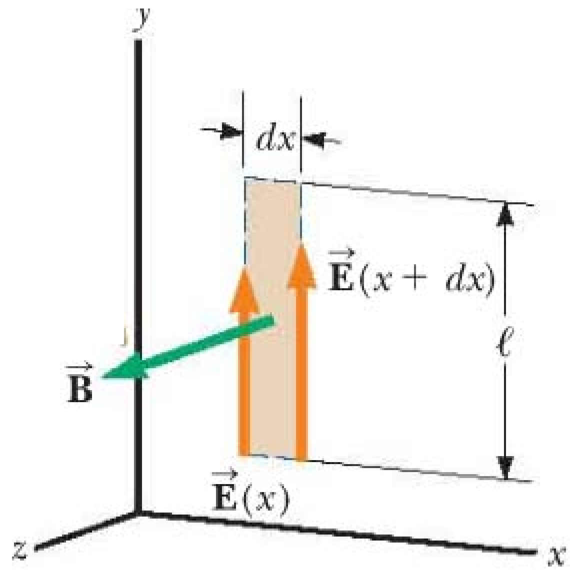


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{E}(x)$ to $\vec{E}(x+dx)$.

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\oiint \vec{B}\cdot d\vec{A}$$

$$\oint \vec{E}\cdot d\vec{s} = E(x+dx)\ell + 0 - E(x)\ell + 0$$

$$= \ell[E(x+dx) - E(x)]$$

$$= \ell\frac{\partial E}{\partial x}\cdot dx$$

$$-\frac{d}{dt}\oiint \vec{B}\cdot d\vec{A} = -\frac{\partial}{\partial t}(B\cdot \ell\, dx)$$

$$= -\ell\frac{\partial B}{\partial t}\cdot dx$$

$$\therefore\ \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

Applying Maxwell's Fourth Equation to Plane Electromagnetic Waves

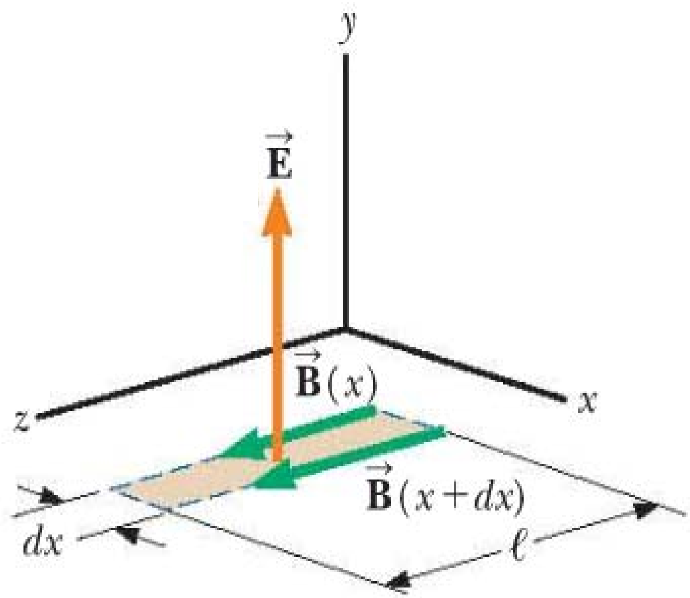


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\vec{B}(x)$ to $\vec{B}(x + dx)$.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I_{in} + \varepsilon_0 \frac{d}{dt} \oiint \vec{E} \cdot d\vec{A}) \qquad (I_{in} = 0)$$

$$\oint \vec{B} \cdot d\vec{s} = B(x)\ell + 0 - B(x+dx)\ell + 0$$

$$= -\ell[B(x+dx) - B(x)]$$

$$= -\ell \frac{\partial B}{\partial x} \cdot dx$$

$$\mu_0 \varepsilon_0 \frac{d}{dt} \oiint \vec{E} \cdot d\vec{A} = \varepsilon_0 \mu_0 \frac{\partial}{\partial t}(E \cdot \ell\, dx)$$

$$= \varepsilon_0 \mu_0 \ell \frac{\partial E}{\partial t} \cdot dx$$

$$\therefore \; -\frac{\partial B}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$