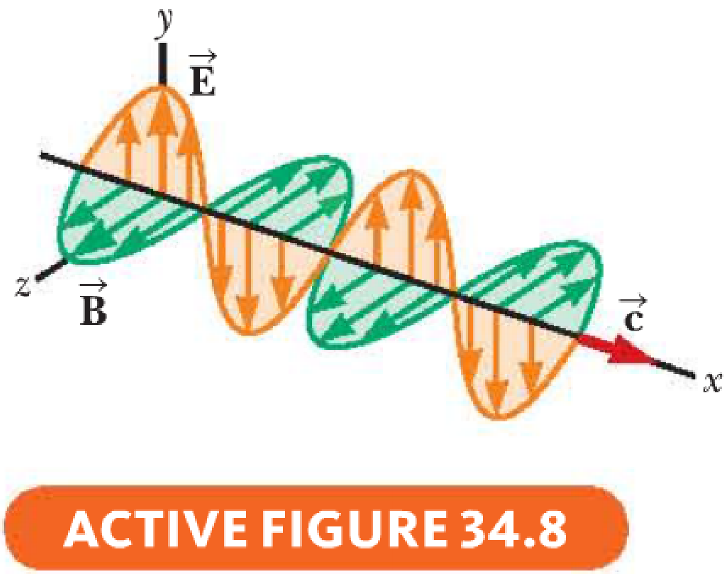
Please Do Course Evaluation

Class 43: Last class – Electromagnetic waves

Congratulation. You have gone a long way!

Electromagnetic plane waves



A sinusoidal electromagnetic wave moves in the positive x direction with a speed c.

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} \quad \text{and} \quad \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = \frac{\partial^2 B}{\partial x^2}$$

Sinusoidal solution :

$$E = E_{max} \cos(kx - \omega t)$$

$$B = B_{max} \cos(kx - \omega t)$$

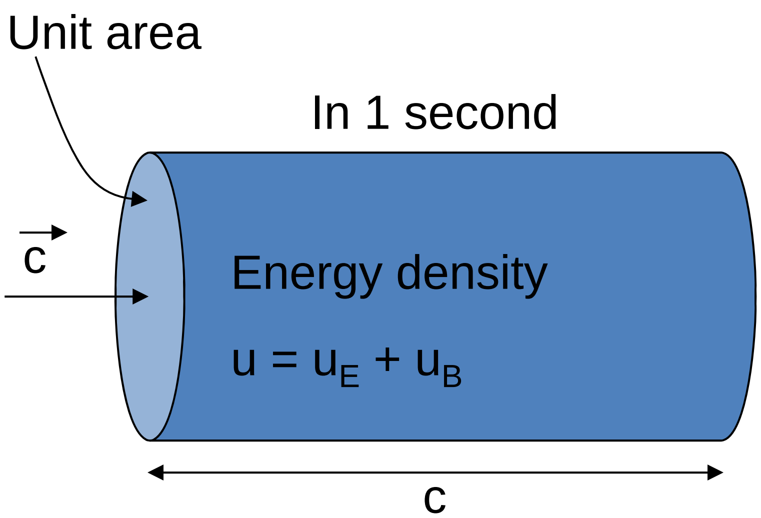
In here : $k = \dfrac{2\pi}{\lambda}$ and $\omega = 2\pi f$

$$\omega = ck, \quad E_{max} = cB_{max}, \text{ and } \quad E = cB$$

$$\Downarrow$$

$$c = f\lambda$$

Intensity



Intensity I is defined as:

$$I = S_{avg}$$

From pointing vector:

$$I = S_{avg}$$

$$\text{But } S = c\left(\frac{1}{\mu_0}B^2\right) \text{ or } c\left(\varepsilon_0 E^2\right)$$

$$\therefore I = c\left(\frac{1}{\mu_0}B^2\right) \text{ or } c\left(\varepsilon_0 E^2\right)$$

$$= c\left(\frac{1}{\mu_0} < B^2 >\right) \text{ or } c\left(\varepsilon_0 < E^2 >\right)$$

$$= c\left(\frac{1}{2\mu_0}B^2_{max}\right) \text{ or } c\left(\frac{\varepsilon_0}{2}E^2_{max}\right)$$

From above picture:

$$I = < \text{energy in the blue cylinder} >$$

$$= c(< u_E > + < u_B >)$$

$$= c\left(\frac{1}{2}\varepsilon_0 < E^2 > + \frac{1}{2\mu_0} < B^2 >\right)$$

$$= c\left(\frac{1}{4}\varepsilon_0 E^2_{max} + \frac{1}{4\mu_0}B^2_{max}\right)$$

$$= c\left(\frac{1}{4}\varepsilon_0 (cB)^2_{max} + \frac{1}{4\mu_0}B^2_{max}\right) \text{ or } c\left(\frac{1}{4}\varepsilon_0 E^2_{max} + \frac{1}{4\mu_0}\left(\frac{E_{max}}{c}\right)^2_{max}\right)$$

$$= c\left(\frac{1}{4\mu_0}B^2_{max} + \frac{1}{4\mu_0}B^2_{max}\right) \text{ or } c\left(\frac{1}{4}\varepsilon_0 E^2_{max} + \frac{1}{4}\varepsilon_0 E^2_{max}\right)$$

$$= c\left(\frac{1}{2\mu_0}B^2_{max}\right) \text{ or } c\left(\frac{1}{2}\varepsilon_0 E^2_{max}\right)$$

Poynting vector

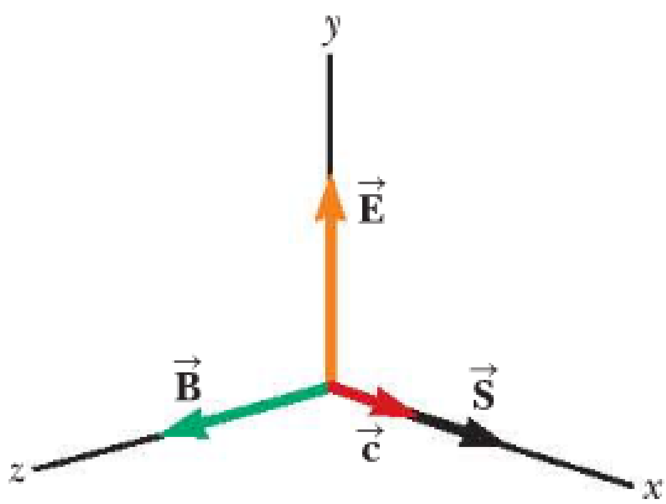


Figure 34.10 The Poynting vector $\vec{S}$ for a plane electromagnetic wave is along the direction of wave propagation.

At any point, knowing $\vec{E}$ and $\vec{B}$, we can define Poynting vector $\vec{S}$ as:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

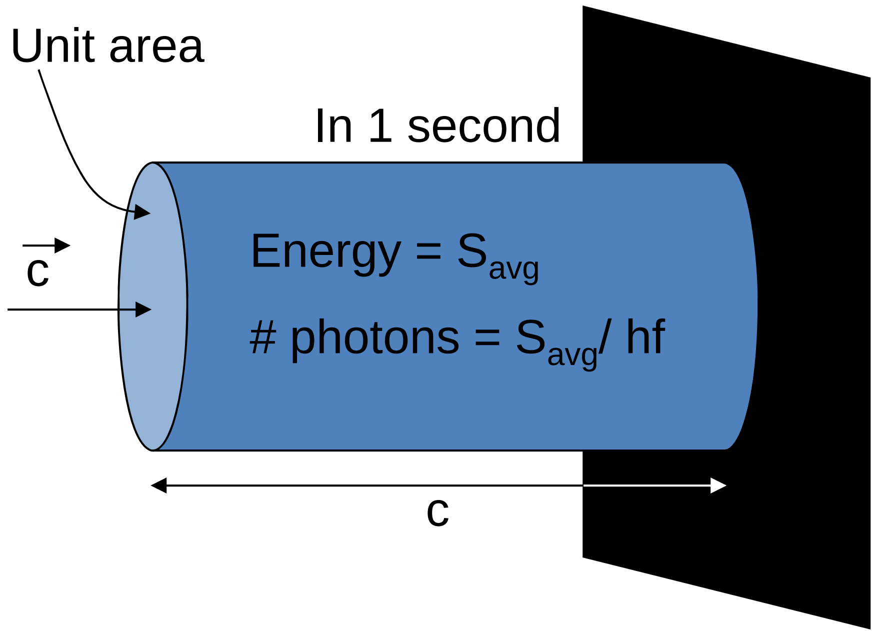
Poynting vector gives the energy passes through a unit surface area perpendicular to the direction of wave propagation. $\vec{S}$ is along the direction of wave propagation and has unit W/m^2.

For plane wave:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \Rightarrow S = \frac{EB}{\mu_0} = \frac{c}{\mu_0}B^2 \text{ or } \frac{1}{c\mu_0}E^2$$

$$\Rightarrow S = c\left(\frac{1}{\mu_0}B^2\right) \text{ or } c\,(\varepsilon_0 E^2)$$

Momentum



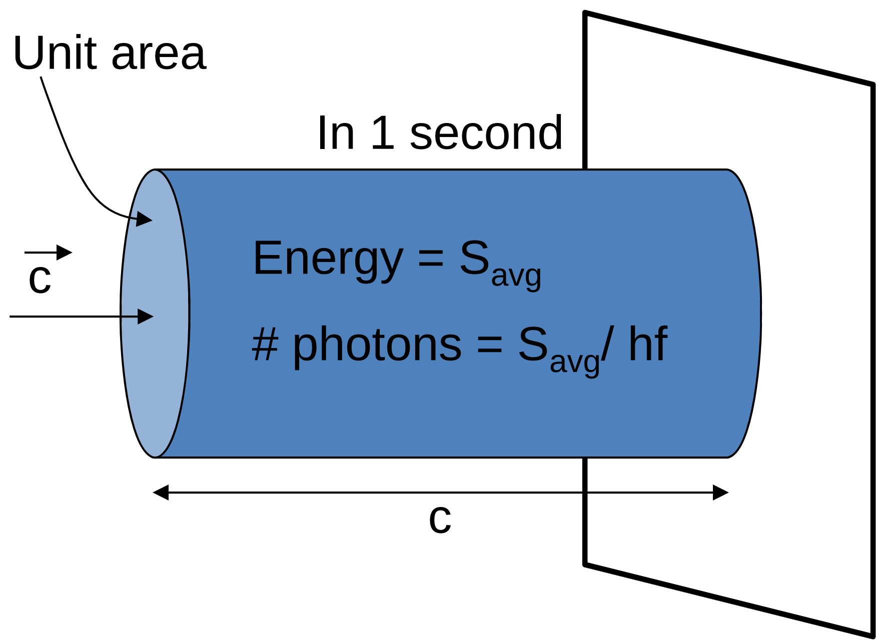
If all these photons are absorbed by the surface:

$$\text{In one second,}\ \ \Delta p = \frac{S_{avg}}{c}$$

$$\therefore F_{\text{unit area}} = \frac{\Delta p}{\Delta t} = \frac{S_{avg}}{c}$$

But this is on unit area

$$\therefore P = \frac{S_{avg}}{c}$$



If all these photons are reflected by the surface:

$$\text{In one second,}\ \ \Delta p = 2 \cdot \frac{S_{avg}}{c}$$

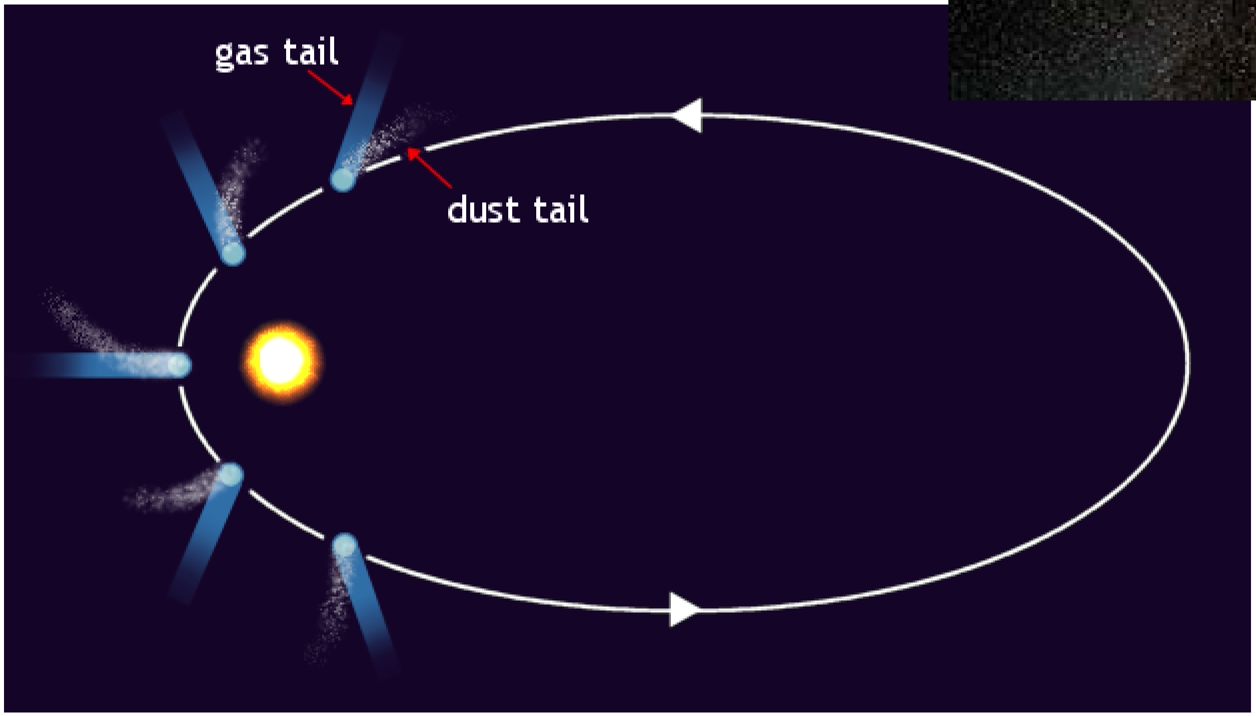
$$\therefore F_{\text{unit area}} = \frac{\Delta p}{\Delta t} = \frac{2S_{avg}}{c}$$

But this is on unit area

$$\therefore P = \frac{2S_{avg}}{c}$$

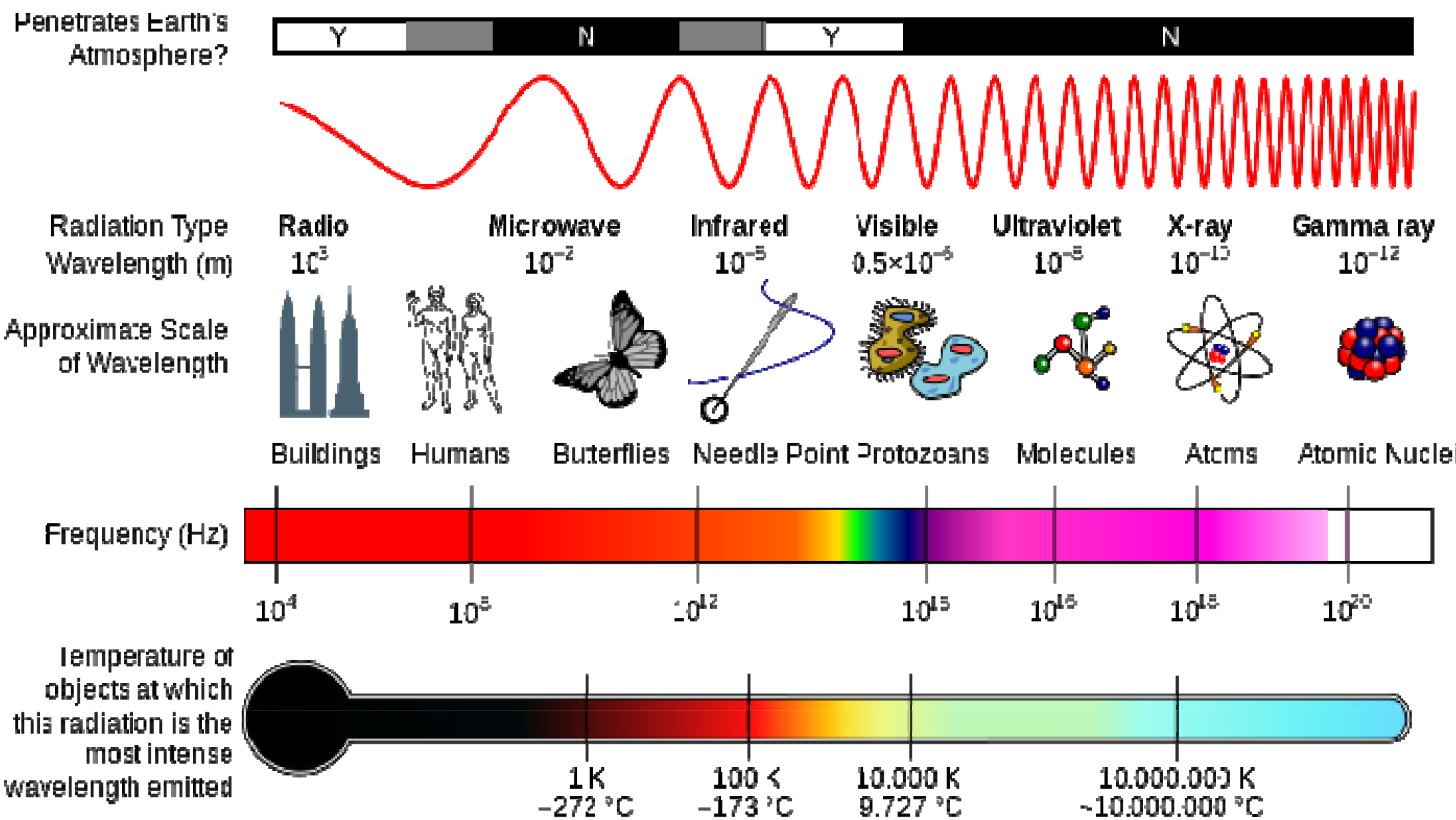
Radiation Pressure

IKAROS



Comet

Electromagnetic spectrum



Why aether?

Newton's Law is "invariant" under Galilean transformation:

Galilean transformation

$$x' = x - vt$$

$$t' = r$$

$$F = m\frac{d^2x}{dt^2} \longrightarrow F = m\frac{d^2x'}{dt'^2}$$

Classical wave equation is "invariant" under Galilean transformation:

Galilean transformation

$$x' = x - vt$$

$$t' = r$$

Does not look like:

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \longrightarrow \frac{\partial^2\psi}{\partial x'^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t'^2}$$

Classical solution:

The wave equation works only in the observing frame where the media is at rest. For light, if the medium is aether, the wave equation will work only in the observing frame where aether is at rest. The problem is: there is no aether!

Special Relativity

Einstein's solution (for waves traveling with speed of light only):

Classical wave equation is "invariant" under Lorentz transformation:

Lorentz transformation

$$x' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(x - vt) \quad \text{and} \quad t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{vx}{c^2}\right)$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \longrightarrow \frac{\partial^2\psi}{\partial x'^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t'^2}$$

Newton's Law is "invariant" under Lorentz transformation:

Lorentz transformation

$$x' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(x - vt) \quad \text{and} \quad t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{vx}{c^2}\right)$$

$$F = m\frac{d^2x}{dt^2} \longrightarrow$$

Does not look like:

$$F = m\frac{d^2x'}{dt'^2}$$

But this is okay because Lorentz transformation will give back Galilean transformation when v is small. In other words, Newton's law of motion is still invariant under Lorentz transformation, but only for small v (true for most parts of our daily life).