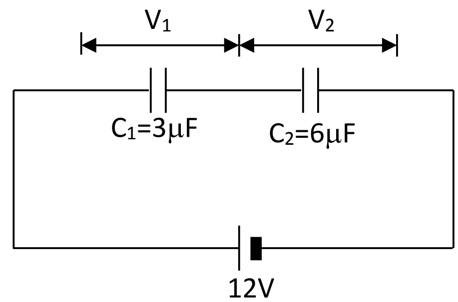
Name: ______________________________ Sec: ___________

PHY 232 Fall 2014 Supplementary Work (will not be collected)

Class 18. Capacitors in series and parallel

Part 1.



(a) What is the charge stored in C_1? What is the charge stored in C_2?

Solution

The first instinct is:

$$C = \frac{Q}{V} \Rightarrow Q = CV \Rightarrow Q_1 = C_1 V_1 \qquad \text{--(*)}$$

While there is nothing wrong with this equation, but we cannot use this here because we do not know what is V_1 yet! So we need some other way to figure out Q_1 or V_1:

Since these two capacitors are in series, so $Q_1=Q_2=Q_{eff}$ and Q_{eff} can be figured out from

$$Q_{eff} = C_{eff} V \text{ with } \frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \Rightarrow C_{eff} = 2\mu F$$

$$\therefore Q_{eff} = (2\mu F)(12) = 24\mu C$$

$$\therefore Q_1 = Q_{eff} = \underline{\underline{24\mu C}} \text{ and } Q_2 = Q_{eff} = \underline{\underline{24\mu C}}$$

(b) What is the voltage V_1 across C_1? What is the voltage V_2 across C_2?

Solution

Now we can use (*) to calculate V_1 because we now know Q_1.

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} \Rightarrow V_1 = \frac{Q_1}{C_1} = \frac{24}{3} = \underline{\underline{8V}}$$

$$\text{Similarly, } V_2 = \frac{Q_2}{C_2} = \frac{24}{6} = \underline{\underline{4V}}$$

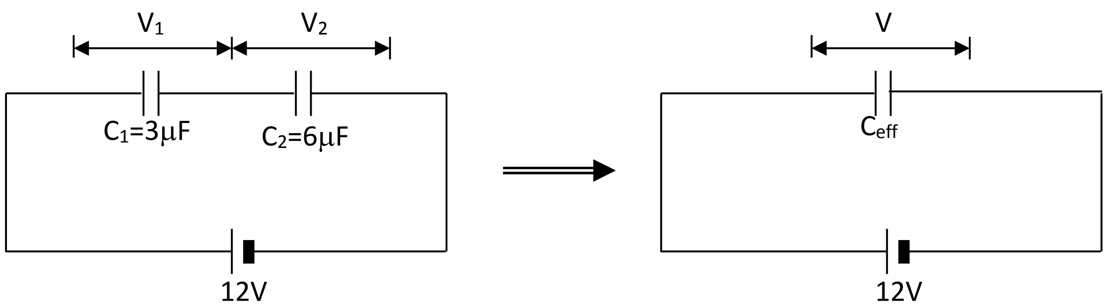
Note that $8V + 4V = 12V$, voltage of the battery.

(c) What is the energy stored in C_1? What is the energy stored in C_2?

Solution

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(3\mu)(8)^2 = \underline{\underline{96\mu J}}$$

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(6\mu)(4)^2 = \underline{\underline{48\mu J}}$$



(d) If the two capacitors are replaced with one, what should be the effective capacitance C_{eff} of the replacement?

Solution

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \Rightarrow C_{eff} = \underline{\underline{2\mu F}}$$

(e) What is the charge stored in C_{eff}? How is this answer compared with that of part (a)?

Solution

$$Q_{eff} = C_{eff}V \text{ with } C_{eff} = 2\mu F$$
$$\therefore Q_{eff} = (2\mu F)(12) = \underline{\underline{24\mu C}}$$

Q_{eff} is the same as Q_1 and Q_2.

(f) What is the voltage across C_{eff}? How is this answer compared with that of part (b)?

Solution

$$Q_{eff} = C_{eff}V \Rightarrow V = \frac{Q_{eff}}{C_{eff}} = \frac{24\mu}{2\mu} = \underline{\underline{12V}}$$

This equals to the sum of $V_1$and V_2. This is reasonable as C_1 and C_2 are connected in series to form C_{eff}.

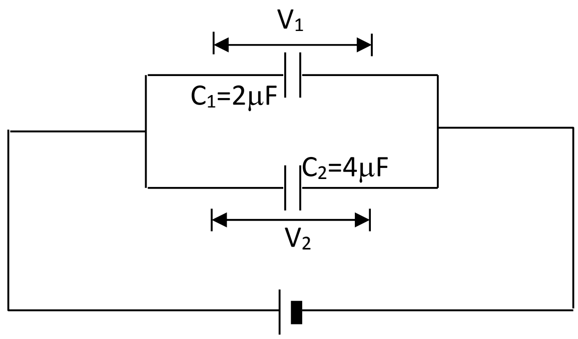
(g) What is the energy stored in C_{eff}? How is this answer compared with that of part (c)?

Solution

$$U_{eff} = \frac{1}{2}C_{eff}V_{eff}^{\ 2} = \frac{1}{2}(2\mu)(12)^2 = \underline{\underline{144\mu J}}$$

This equals to U_1+U_2.

Part 2.



(a) What is the charge stored in C_1? What is the charge stored in C_2?

Solution

Note that $V_1=V_2=12V$ (we are already answering part (b)!)

$$C=\frac{Q}{V} \Rightarrow Q=CV \Rightarrow Q_1=C_1V_1 \Rightarrow Q_1=(2\mu)(12)=\underline{\underline{24\mu C}}$$

$$\text{Similarly,} Q_2=C_2V_2 \Rightarrow Q_1=(4\mu)(12)=\underline{\underline{48\mu C}}$$

(b) What is the voltage V_1 across C_1? What is the voltage V_2 across C_2?

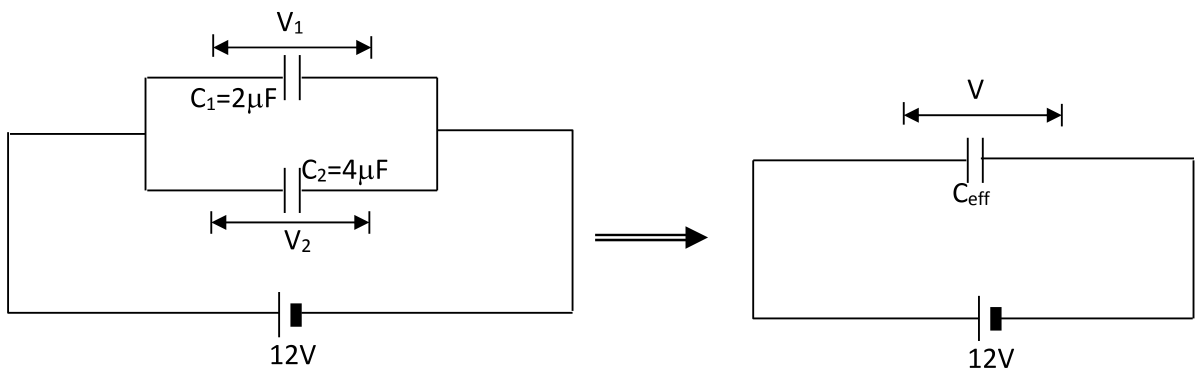
Solution

$$V_1=V_2=\underline{\underline{12V}}$$

(c) What is the energy stored in C_1? What is the energy stored in C_2?

$$U_1=\frac{1}{2}C_1V_1^2=\frac{1}{2}(2\mu)(12)^2=\underline{\underline{144\mu J}}$$

$$U_2=\frac{1}{2}C_2V_2^2=\frac{1}{2}(4\mu)(12)^2=\underline{\underline{288\mu J}}$$



(d) If the two capacitors are replaced with one, what should be the effective capacitance C_{eff} of the replacement?

Solution

$$C_{eff} = C_1 + C_2 = 2\mu + 4\mu = \underline{\underline{6\mu F}}$$

(e) What is the charge stored in C_{eff}? How is this answer compared with that of part (a)?

Solution

$$Q_{eff} = C_{eff} V \text{ with } C_{eff} = 6\mu F$$
$$\therefore Q_{eff} = (6\mu F)(12) = \underline{\underline{72\mu C}}$$

This is the sum of Q_1 and Q_2 (24μC + 48μC = 72 μC)

(f) What is the voltage across C_{eff}? How is this answer compared with that of part (b)?

Solution

This should be the same as V_1 and V_2, 12V.

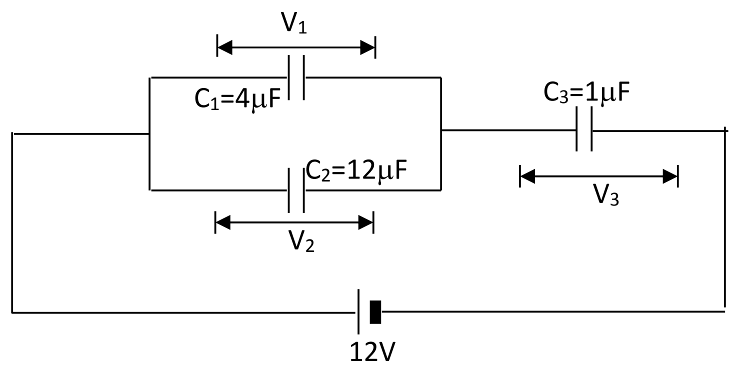
(g) What is the energy stored in C_{eff}? How is this answer compared with that of part (c)?

Solution

$$U_{eff} = \frac{1}{2} C_{eff} V_{eff}^{\ 2} = \frac{1}{2}(6\mu)(12)^2 = \underline{\underline{432\mu J}}$$

This equals to U_1+U_2, 144μJ+288μJ =432μJ.

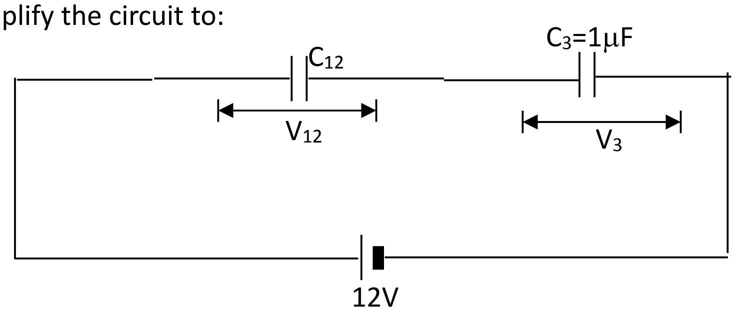
Part 3.



(a) What is the charge stored in C_1? What is the charge stored in C_2? What is the charge stored in C_3 ?

Partial Solution

We first simplify the circuit to:



C_{12} is the effective capacitance of C_1 and C_2 in parallel.

$$C_{12} = C_1 + C_2 = 4\mu + 12\mu = 16\mu F$$

Since C_{12} and C_3 are now in series, so $Q_{12}=Q_3=Q_{eff}$ and Q_{eff} can be figured out from the effective capacitance of C_{12} and C_3:

$$Q_{eff} = C_{eff} V \text{ with } \frac{1}{C_{eff}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{16} + \frac{1}{1} = \frac{1}{2} \Rightarrow C_{eff} = 0.9412\mu F$$

$$\therefore Q_{eff} = (0.9412\mu F)(12) = 11.294\mu C$$

$$\therefore Q_3 = Q_{eff} = \underline{\underline{11.294\mu C}}$$

It is easier to figure out V_1 and V_2 first before Q_1 and Q_2, so we proceed to part (b) first then come back for Q_1 and Q_2.

(b) What is the voltage V_1 across C_1? What is the voltage V_2 across C_2? What is the voltage V_2 across C_3?

Solution

Note that we now know Q_3.

$$C=\frac{Q}{V}\Rightarrow V=\frac{Q}{C}\Rightarrow V_3=\frac{Q_3}{C_3}=\frac{11.294}{1}=\underline{\underline{11.294V}}$$

Since C_{12} is connected to C_3 in series,

$$\therefore V_{12}+V_3=V\Rightarrow V_{12}+11.294=12\Rightarrow V_{12}=0.706V$$

$$\therefore V_1=V_{12}=\underline{\underline{0.706V}}$$

$$V_2=V_{12}=\underline{\underline{0.706V}}$$

Now we come back to complete the solution of part (a):

(a) What is the charge stored in C_1? What is the charge stored in C_2? What is the charge stored in C_3 ?

Completing Solution

$$C=\frac{Q}{V}\Rightarrow Q=CV\Rightarrow Q_1=C_1V_1\Rightarrow Q_1=(4\mu)(0.706)=\underline{\underline{2.824\mu C}}$$

$$\text{Similarly,}Q_2=C_2V_2\Rightarrow Q_1=(4\mu)(0.706)=\underline{\underline{8.471\mu C}}$$

Note that $Q_1+Q_2=2.824\mu C+8.471\mu C=11.295\mu C=Q_{12}=Q_3$.

(c) What is the energy stored in C_1? What is the energy stored in C_2? What is the energy stored in C_3?

Solution

$$U_1=\frac{1}{2}C_1V_1^2=\frac{1}{2}(4\mu)(0.706)^2=\underline{\underline{0.9965\mu J}}$$

$$U_2=\frac{1}{2}C_2V_2^2=\frac{1}{2}(12\mu)0.706)^2=\underline{\underline{2.9896\mu J}}$$

$$U_3=\frac{1}{2}C_3V_3^2=\frac{1}{2}(1\mu)(11.294)^2=\underline{\underline{63.777\mu J}}$$