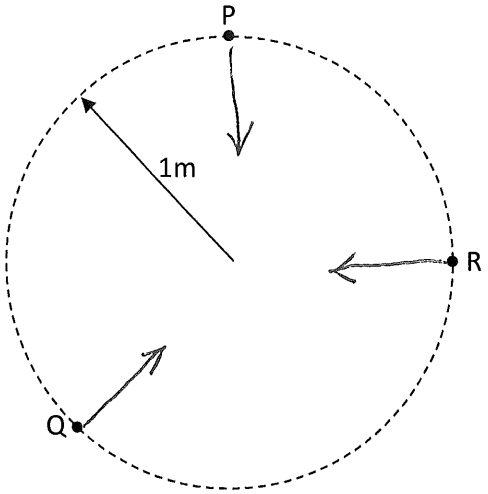
PHY 232 Fall 2017 Example

Class 1. Circular motion and gravitational law

You see a particle moving in a circle with a constant speed of 2 ms^{-1}. Radius of the circular orbit is 1m.



(a) Sketch the acceleration direction at points P, Q, and R in the above figure. Do you need to know whether the circular motion is in clockwise or anticlockwise direction?

(b) What is the magnitude of the acceleration?

$$a_c = \frac{v^2}{r} = \frac{2^2}{1} = 4\ m/s^2.$$

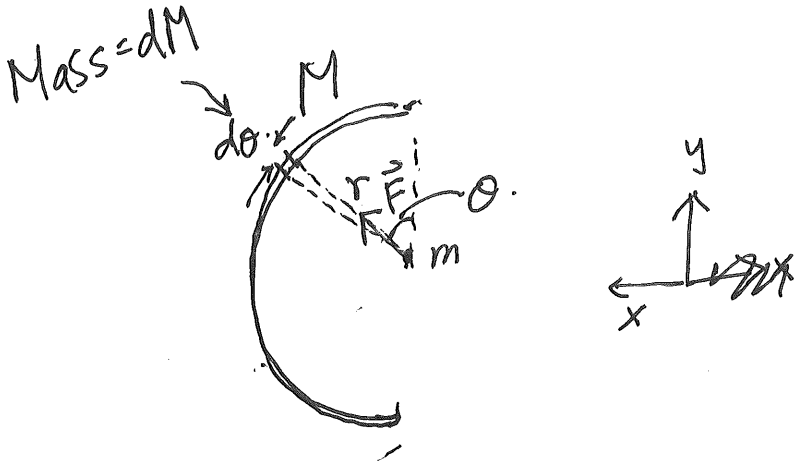
(c) Calculate angular velocity ω, frequency f, and period T.

$$v = r\omega \Rightarrow \omega = \frac{v}{r} = \frac{2}{1} = 2\ rad/s.$$

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{2}{2\pi} = 0.318\ s^{-1}$$

or $0.318\ Hz$.

$$T = \frac{1}{f} = 3.14\ s.$$



$$dM = \frac{M}{\pi r} \cdot r\,d\theta.$$

$$dF_x = \frac{dM\,m}{r^2}\sin\theta.$$

$$F_x = \int_0^{\pi} \left(\frac{M}{\pi r} \cdot r\,d\theta\right) \frac{m}{r^2}\sin\theta.$$

$$= \int_0^{\pi} \frac{Mm}{\pi r^2}\sin\theta\,d\theta.$$

$$= \frac{Mm}{\pi r^2}\int_0^{\pi} \sin\theta\,d\theta$$

$$= \frac{Mm}{\pi r^2}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{2Mm}{\pi r^2}.$$

$$F_y = 0.$$

$$\therefore \vec{F} = \underline{\underline{\frac{2mM}{\pi r^2}\,\hat{i}}}\,.$$