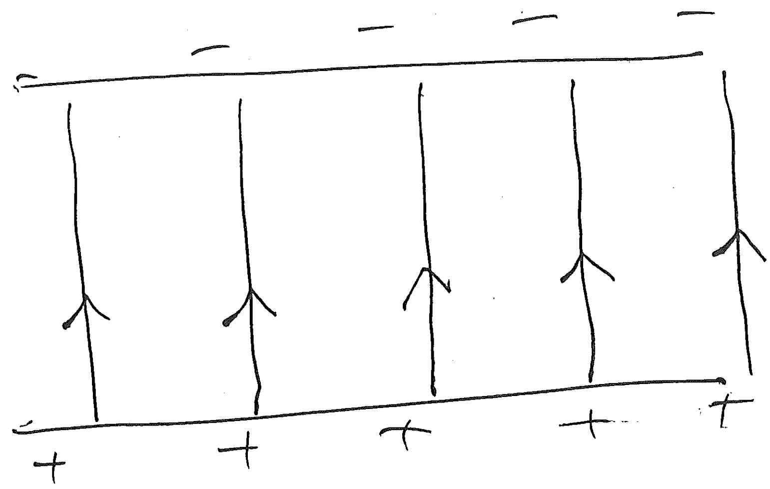
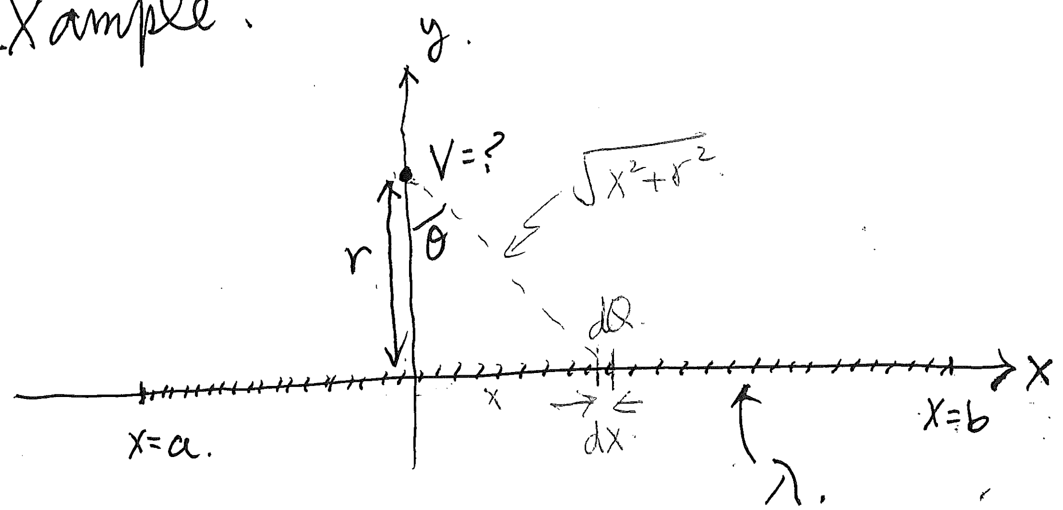


From last class:

Example.



Understand!?

$$\therefore\ dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{\sqrt{x^2+r^2}}$$

$$V = \int \frac{1}{4\pi\varepsilon_0}\frac{dQ}{\sqrt{x^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\int_a^b \frac{\lambda\, dx}{\sqrt{x^2+r^2}}$$

$\cancel{x=}$ $x = r\tan\theta.$ $\quad dx = \dfrac{r}{\cos^2\theta}\, d\theta.$

$\left(\because \dfrac{d(\tan\theta)}{d\theta} = \dfrac{1}{\cos^2\theta}\right).$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_{x=a}^{x=b} \frac{r/\cos^2\theta\, d\theta}{r/\cos\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{x=a}^{x=b} \frac{d\theta}{\cos\theta}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int \frac{\cos\theta}{\cos^2\theta}\, d\theta.$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int \frac{\cos\theta}{1-\sin^2\theta}\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int \frac{du}{1-u^2}$$

$\cancel{\cos\theta}$ $\sin\theta = u.$
$\cos\theta\, d\theta = du.$

$(1-u)(1+u)$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int \frac{1}{2}\left(\frac{1}{1-u} + \frac{1}{1+u}\right) du$$

$\left[\dfrac{1}{1-u} + \dfrac{1}{1+u} = \dfrac{2}{(1-u)(1+u)}\right.$

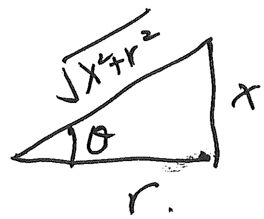
$\left.\dfrac{1}{1-u^2} = \dfrac{1}{2}\left(\dfrac{1}{1-u} + \dfrac{1}{1+u}\right)\right]$

$$= \frac{\lambda}{8\pi\varepsilon_0}\left[-\ln(1-u) + \ln(1+u)\right].$$

$$= \frac{\lambda}{8\pi\varepsilon_0}\ln\frac{1+u}{1-u}.$$

$$= \frac{\lambda}{8\pi\varepsilon_0}\ln\frac{1+\sin\theta}{1-\sin\theta}.$$

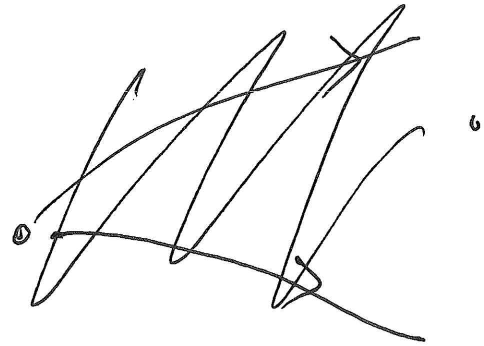
$x = r\tan\theta.$

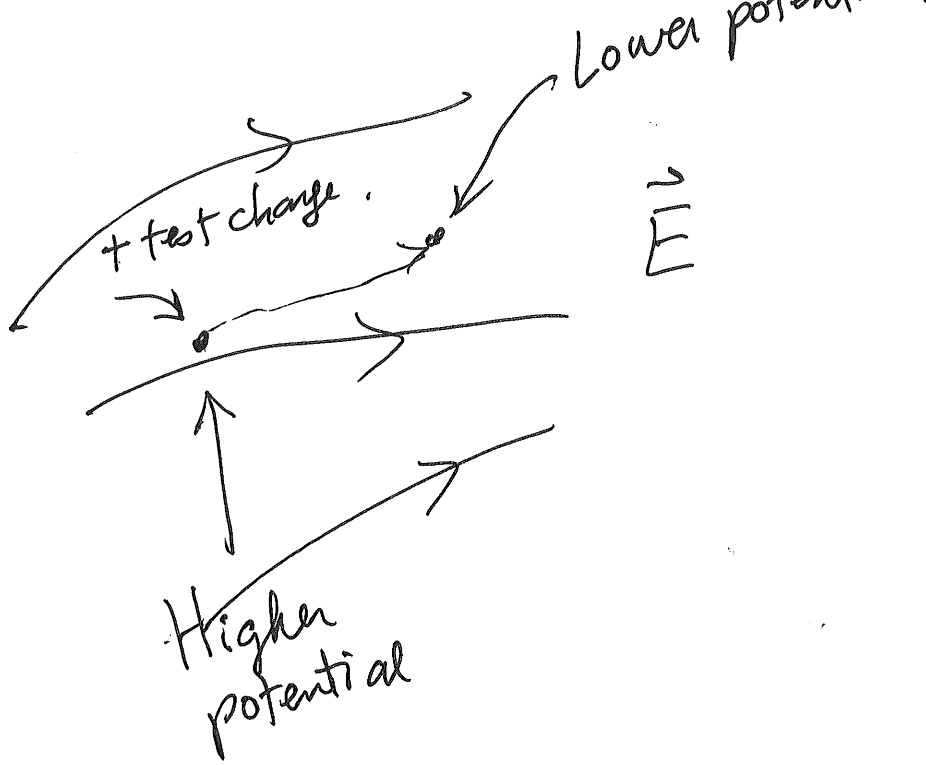


$$= \frac{\lambda}{8\pi\varepsilon_0}\ln\frac{1+\frac{x}{\sqrt{x^2+r^2}}}{1-\frac{x}{\sqrt{x^2+r^2}}}.$$

$$= \frac{\lambda}{8\pi\varepsilon_0}\ln\frac{\sqrt{x^2+r^2}+x}{\sqrt{x^2+r^2}-x}.\Bigg]_a^b.$$

$$= \frac{\lambda}{8\pi\varepsilon_0}\ln\left[\frac{(\sqrt{r^2+b^2}+b)(\sqrt{r^2+a^2}-a)}{(\sqrt{r^2+a^2}+a)(\sqrt{r^2+b^2}-b)}\right] //$$





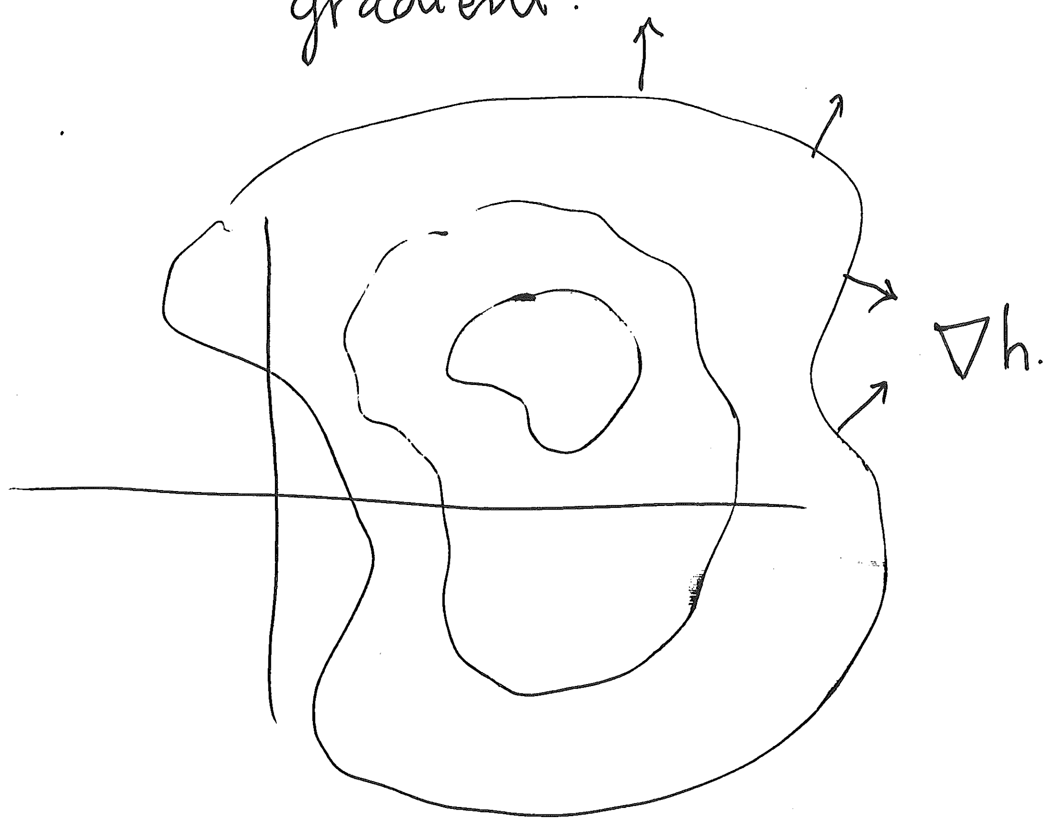
$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right).$$

$$\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

$$\vec{E} = -\nabla V.$$

$$E = -\nabla V \text{ or } -\text{grad } V.$$

gradient.



Ex. If $V = 3x^2yz^3$

$$\vec{E} = -\left(\frac{\partial}{\partial x}3x^2y\,,\ \frac{\partial}{\partial y}3x^2yz^3,\ \frac{\partial}{\partial z}3x^2yz^3\right).$$

$$= -\left(6xyz^3,\ 3x^2z^3,\ 9x^2yz^2\right).$$

$$= -6xyz^3\hat{i} - 3x^2z^3\hat{j} - 9x^2yz^2\hat{k}.$$