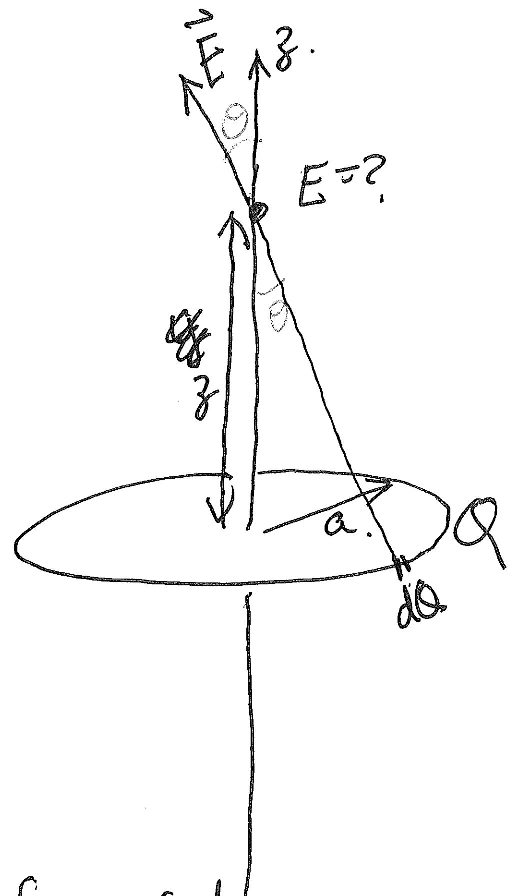


$$\cos\theta = \frac{z}{\sqrt{z^2+a^2}}$$

ONLY for point on z-axis:

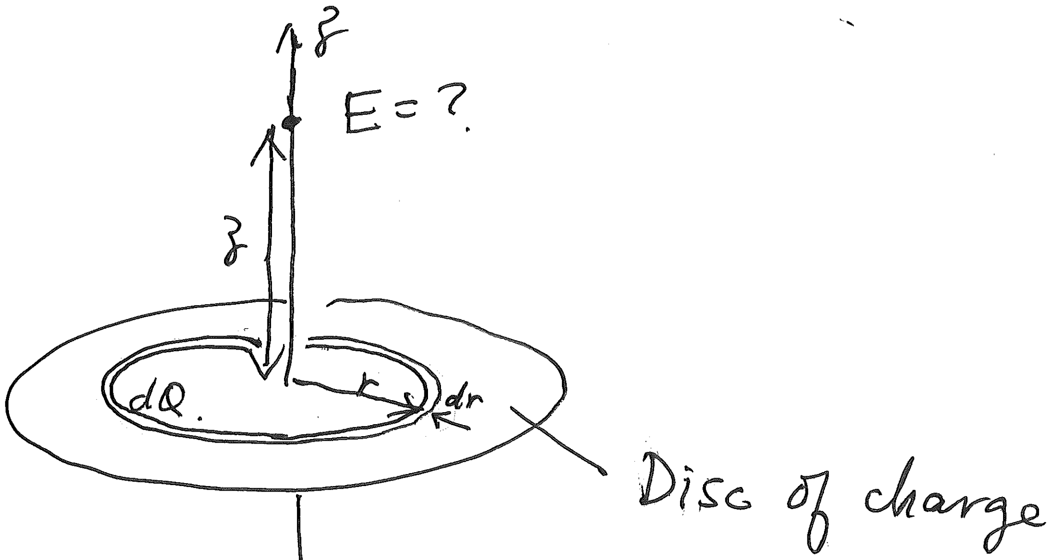
$$E_x = E_y = 0$$

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{a^2+z^2}\cos\theta \quad (r^2=a^2+z^2).$$

$$\therefore E_z = \int dE_z = \frac{1\cdot\cos\theta}{4\pi\varepsilon_0(a^2+z^2)}\int dQ$$

$$= \frac{Q\cos\theta}{4\pi\varepsilon_0(a^2+z^2)}$$

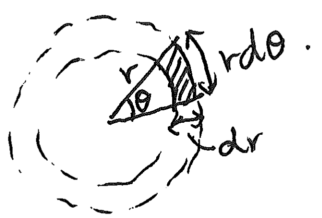
$$= \frac{Qz}{4\pi\varepsilon_0(a^2+z^2)^{3/2}}$$



dE due to the infinitesimal ring

$$= \frac{1}{4\pi\varepsilon_0}\frac{z\,dQ}{(z^2+r^2)^{3/2}}$$

~~To~~ $$E = \frac{1}{4\pi\varepsilon_0}\int \frac{z\,dQ}{(z^2+r^2)^{3/2}}$$



$$dQ = \sigma\, dA$$

$$dA = \int_0^{2\pi} r\,d\theta\,dr = 2\pi r\,dr \qquad = \sigma\, \underbrace{2\pi r\,dr}_{dA}$$

(to be continued)

$$E = \frac{z}{4\pi\varepsilon_0} \int \frac{\sigma \cdot 2\pi r\,dr}{(z^2+r^2)^{3/2}}.$$

$$= \frac{2\pi z \sigma}{4\pi\varepsilon_0} \int \frac{r\,dr}{(z^2+r^2)^{3/2}}.$$

m m m

$$= \frac{z}{2\varepsilon_0} \cdot \underbrace{\frac{Q}{\pi R^2}}_{\sigma} \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}.$$

m^2 m^3

$$\left(\cancel{u = z^2 + r^2}\right.$$

du =

Let $r^2 = u$. $2r\,dr = du$, $\left(\begin{matrix} u = z^2+r^2 \\ du = 2r\,dr \end{matrix}\right)$.

also good substitution.

Dimension (units) check

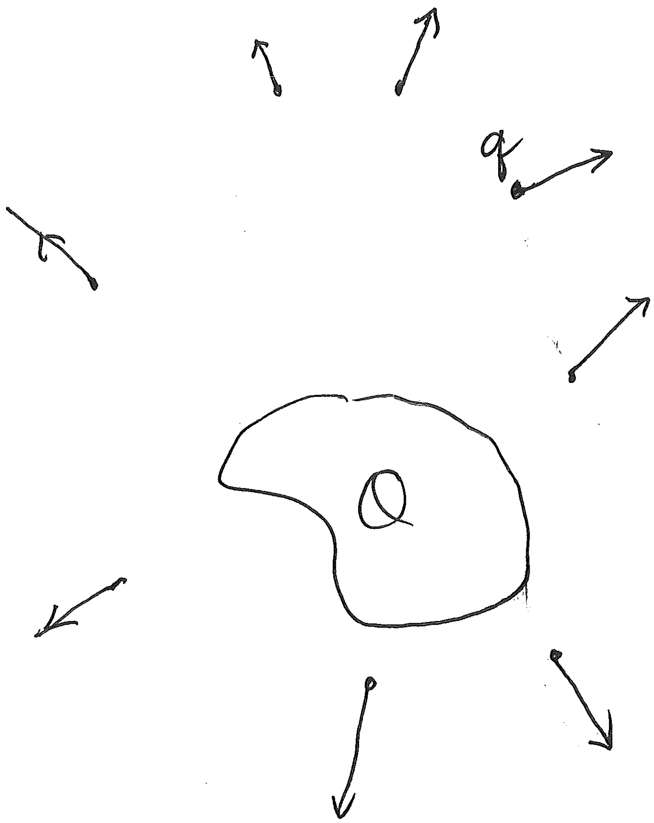
$$E = \frac{z}{2\varepsilon_0}\frac{Q}{\pi R^2} \int_0^{R^2} \frac{du/2}{(z^2+u)^{3/2}}.$$

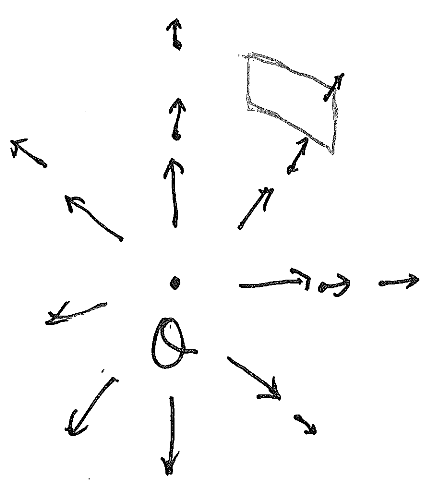
$$= \frac{z}{2\varepsilon_0}\frac{Q}{\pi R^2}\frac{1}{2} \cdot \left[2 \cdot \frac{-1}{(z^2+u)^{1/2}}\right]_0^{R^2}.$$

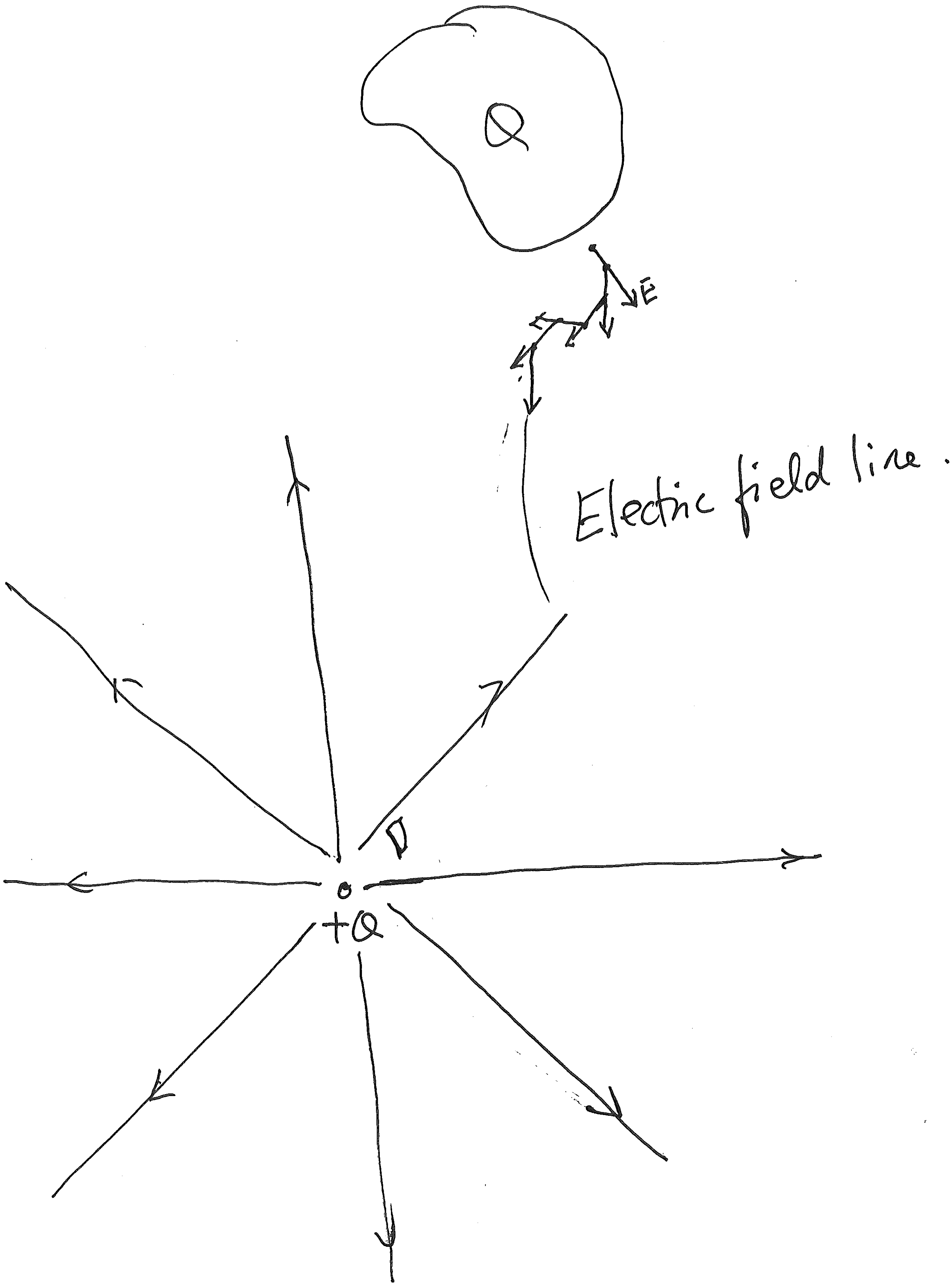
$$= \frac{z}{2\varepsilon_0}\frac{Q}{\pi R^2}\left[\left(-\frac{1}{\sqrt{z^2+R^2}}\right) - \left(-\frac{1}{z}\right)\right].$$

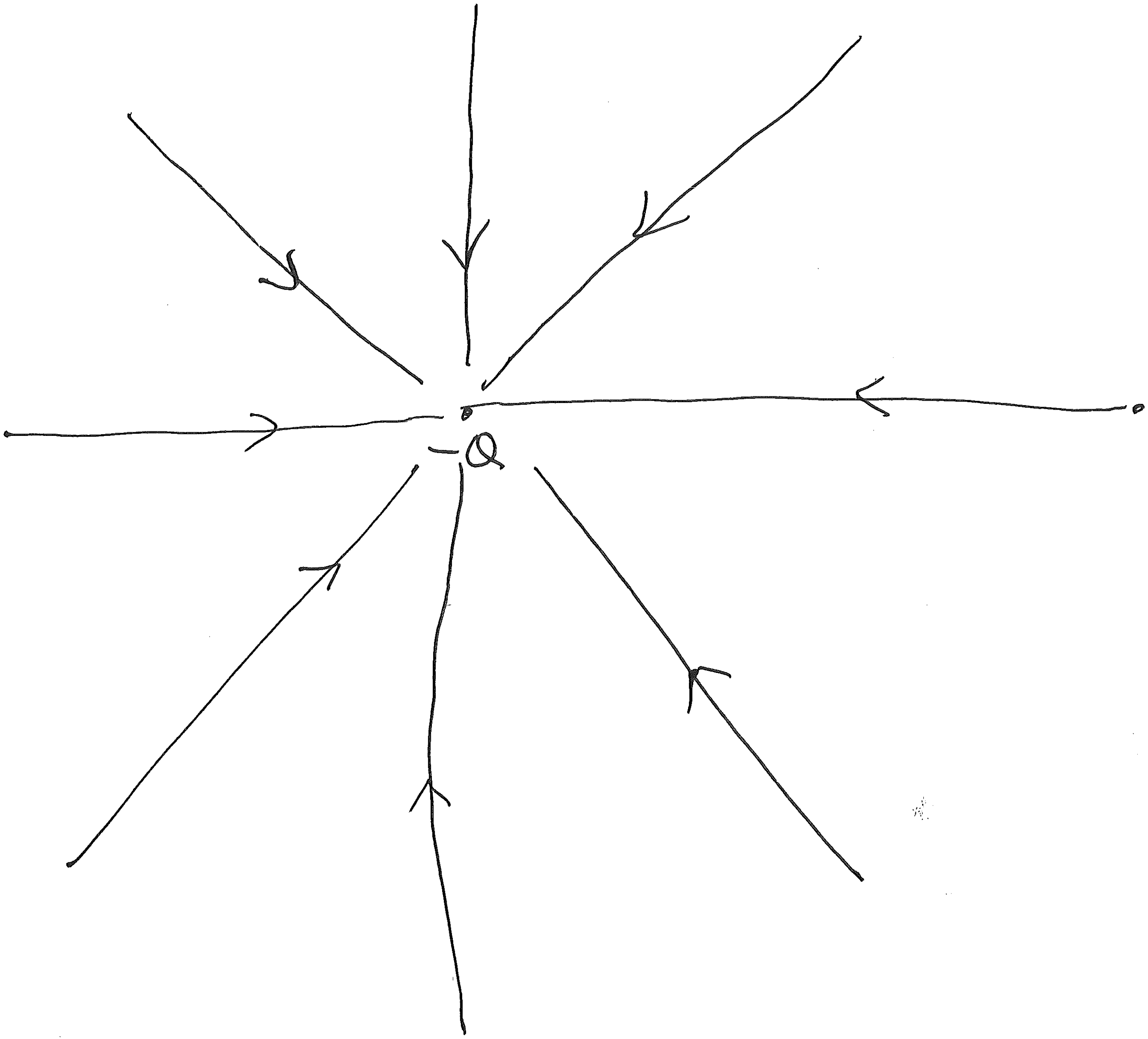
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad /m^2$$

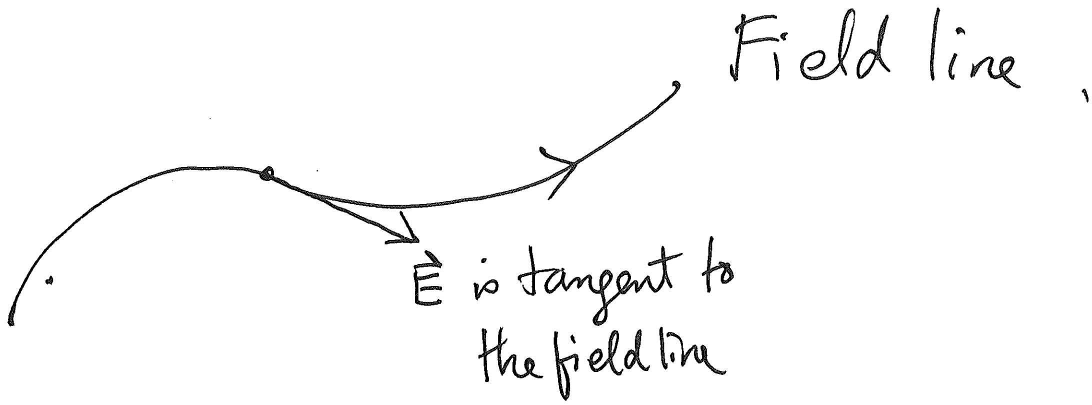
$$= \frac{z}{2\varepsilon_0}\frac{Q}{\pi R^2}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

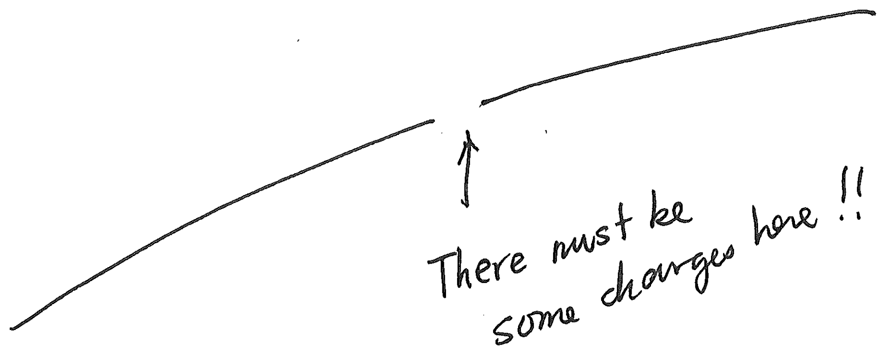

q
Q


Q


Q
E
Electric field line.
+Q


-Q


Field line
$\vec{E}$ is tangent to
the field line



Electric flux

$$\Phi_E = E \cdot A \cdot \cos\theta$$

$$= \vec{E} \cdot \vec{A}$$