$$\varepsilon_0 \int \vec{E} \cdot d\vec{A} = Q_{in} \Rightarrow \varepsilon_0 \int E \cos\theta \, dA = Q_{in}$$

↑

Difficult !!

IF we can find a surface :

1). $\vec{E} \parallel d\vec{A}$ ($\vec{E}$ perpendicular to surface)

AND

$$\vec{E} \cdot d\vec{A} = E\,dA \cos 0^\circ = E\,dA$$

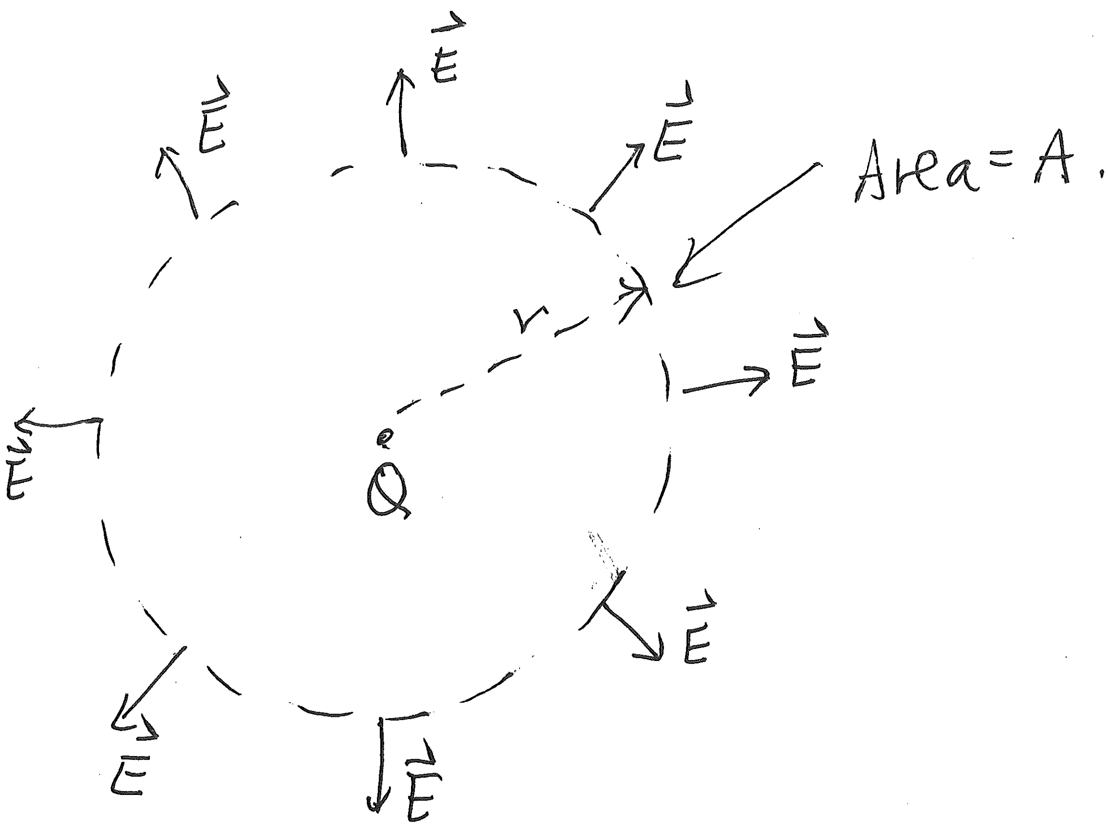
2) E is a constant on that surface.

$$\oint \vec{E} \cdot d\vec{A} = \oint E\,dA \quad (\cos 0^\circ = 1)$$

$$= E \oint dA$$

$$= EA$$

$\varepsilon_0 E A = Q_{in} \Rightarrow$ E field on that surface $= \dfrac{Q_{in}}{\varepsilon_0 A}$.



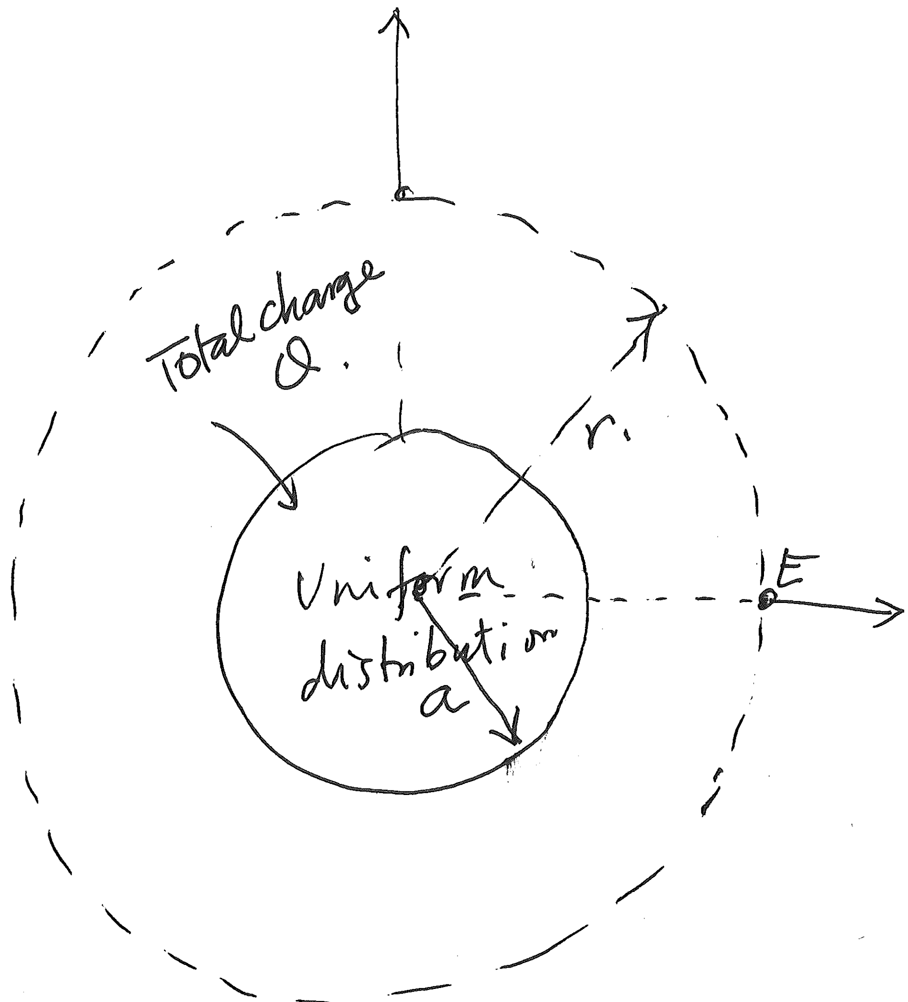
Volume of a sphere $= \frac{4}{3}\pi R^3$

Surface area of a sphere $= 4\pi R^2$.

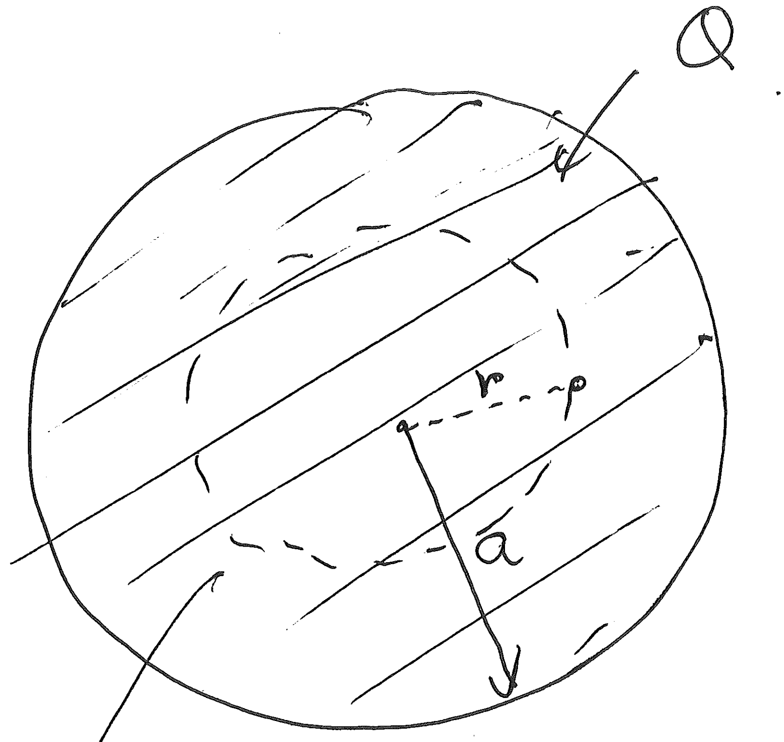
$$\therefore \varepsilon_0 E A = Q$$

$$\varepsilon_0 E \cdot 4\pi r^2 = Q.$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$



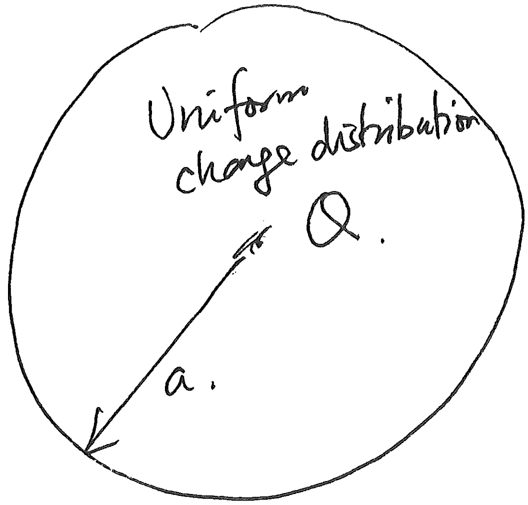
$$\varepsilon_0 E\, 4\pi r^2 = \rho \cdot \text{Volume of}$$
"dotted sphere"

$$= \frac{Q}{\frac{4}{3}\pi a^3} \cdot \frac{4}{3}\pi r^3$$

$$= Q\frac{r^3}{a^3}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0 r^2} \cdot Q\frac{r^3}{a^3}$$

$$= \frac{Qr}{4\pi\varepsilon_0 a^3} \quad //$$



Outside: Electric field for $r > a = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$.

Inside: Electric field for $r < a = \frac{Qr}{4\pi\varepsilon_0 a^3}$.