Name:________________________________

University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2002
Final Examination

Answer all questions. Write down all work in detail.
Time allowed: 120 minutes
Merry Christmas and Happy New Year!

1. A particle of mass m is confined to a one dimensional region $0 \le x \le a$ with the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < a \\ \infty & x > 0 \end{cases}$$

At t=0 its normalized wave function is

$$\Psi(x, t = 0) = \sqrt{\frac{8}{5a}}\left[1 + \cos\left(\frac{\pi x}{a}\right)\right]\sin\left(\frac{\pi x}{a}\right)$$

(a) (8 points)
What is the wave function at a later time $t = t_0$? The Schroedinger eigenfunctions and eigenvalues for the above potential are given as

$$\Psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \quad ; \quad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

(b) (9 points)

What is the average energy of the system at $t=0$ and at $t=t_0$?

c. (8 points)

What is the probability that the particle is found in the left half of the box (i.e., in the region $0 \le x \le a/2$) at $t=t_0$?

2. The time independent normalized wave function for an electron in a hydrogen atom is

$$\psi(\vec{r}) = \frac{1}{\sqrt{10}}(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1})$$

where the subscripts are values of the quantum number *n, l, m*.

(a) (7 points)
Calculate <L^2> and <L$_z$> for the electron.

(b) (7 points)
The first few radial functions R_{nl} for hydrogen atom is given as

$$R_{10}(r) = 2a_0^{-3/2}e^{-r/a_0}$$

$$R_{20}(r) = 2(2a_0)^{-3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}$$

$$R_{21}(r) = \frac{(2a_0)^{-3/2}}{\sqrt{3}}\frac{r}{a_0}e^{-r/2a_0}$$

where a_0 is the Bohr radius, a constant.
Calculate the probability of finding the electron within a distance R from the proton. You can assume R<< a_0.

(c) (4 points)

Multiple choices. Circle the correct answer.

The ground state energy of hydrogen atom is

A. 0 eV
B. -13.6 eV
C. -0.53 eV
D. 13.6 eV
E. 0.53 eV

How does E_n depend on n:

A. $E_n \sim n^2$

B. $E_n \sim n$

C. $E_n \sim 1/n$

D. $E_n \sim 1/n^2$

E. $E_n \sim n\hbar\omega$

(d) (7 points)

Calculate <E> for the electron. (Hint: use results from part (c) above).

3. A particle in a central potential has an orbital angular momentum L=2← and a spin S = 1←. The spin-orbit interaction gives rise to the Hamiltonian H= A **L•S** where A is a constant. Let **J** = **L**+**S**.

(a) (5 points)
Altogether how many eigenstates are there?

(b) (5 points)
What are the quantum numbers that determine the eigenstates of the Hamiltonian?

(c) (5 points)
Write the Hamiltonian in terms of J^2, L^2, and S^2.

(d) (10 points)
Determine all possible energy eigenvalues. Determine the corresponding degeneracy for each of these eigenvalues.