University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2002
Test 1

Answer all questions. Write down all work in detail.
Time allowed: 50 minutes
Good luck!

1. Consider a wave packet given by

$$f(x) = \begin{cases} 0 & x < -a/2 \\ A & -a/2 < x < a/2 \\ 0 & x > a/2 \end{cases}$$

(a) (3 points)
Find the value of A to normalize f(x).

(b) (12 points)
Find the form of g(k). Make sure g(k) is normalized also.

(c) (3 points)
Write down f(x,t) as a series expansion of $e^{i(kx-\omega t)}$.

(d) (7 points)
Show that a reasonable definition of Δk for you answer to (b) yields

$$\Delta x \, \Delta k > 1$$

independent of the value of a.

2. A particle of mass m is confined to a one dimensional region $0 \le x \le a$ with the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < a \\ \infty & x > 0 \end{cases}$$

At t=0 its normalized wave function is

$$\Psi(x, t = 0) = \sqrt{\frac{8}{5a}}\left[1 + \cos\left(\frac{\pi x}{a}\right)\right]\sin\left(\frac{\pi x}{a}\right)$$

(a) (8 points)
What is the wave function at a later time $t = t_0$? The Schroedinger eigenfunctions and eigenvalues for the above potential are given as

$$\Psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \quad ; \; E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

(b) (9 points)
What is the average energy of the syatem at t=0 and at $t=t_0$?

c. (8 points)
What is the probability that the particle is found in the left half of the box (i.e., in the region $0 \leq x \leq a/2$) at $t=t_0$?