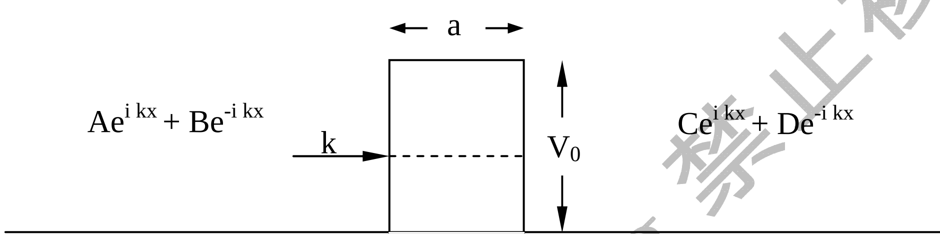
University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2002
Test 2

Answer all questions. Write down all work in detail.
Time allowed: 50 minutes
Show all work clearly. Add extra papers if needed. Good luck!

1. Consider the finite potential barrier shown in the figure:



The scattering matrix is given as:

$$\begin{pmatrix} C \\ B \end{pmatrix} = S \begin{pmatrix} A \\ D \end{pmatrix}$$

$$S = \frac{e^{-2ika}}{\cosh 2\kappa a + \frac{i\varepsilon}{2}\sinh 2\kappa a} \begin{pmatrix} 1 & -\frac{i\eta}{2}\sinh 2\kappa a \\ -\frac{i\eta}{2}\sinh 2\kappa a & 1 \end{pmatrix}$$

where

$$\varepsilon = \frac{\kappa}{k} - \frac{k}{\kappa} \text{ and } \eta = \frac{\kappa}{k} + \frac{k}{\kappa}; \quad \frac{\hbar^2 k^2}{2m} = E \text{ and } \frac{\hbar^2 \kappa^2}{2m} = V_0 - E$$

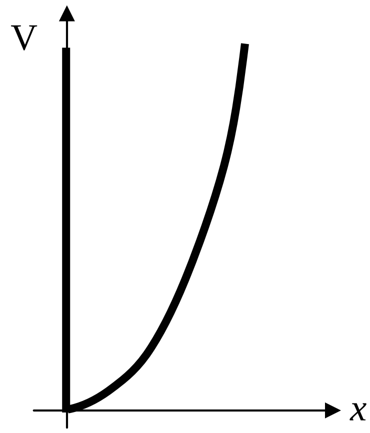
(a) (5 points)
Is S unitary, or Hermitian, or both, or none? Prove your answer.

(b) (10 points)
Calculate the transmission probability $|T|^2$. Further simplify your answer by assuming $|T| << 1$.

(c) (10 points)

An electron with energy E= 1eV is incident upon the potential barrier with V_0 = 2eV. About how wide must the barrier be so that the transmission probability is (i) 10^{-3}, (ii) 10^{-2} ? It is given the mass and charge of electrons are 9.11×10^{-27}g and 1.6×10^{-19} C respectively. Also, $\hbar = 1.055\times10^{-27}$ erg - s.

2. Consider a semi-simple harmonic potential as shown in the figure:



$$V(x) = \begin{cases} \infty & x < 0 \\ \frac{1}{2}kx^2 & x \geq 0 \end{cases}$$

(a) (7 points)
Which of the following CANNOT be an eigenfunction (for $x \geq 0$)of this potential? Explain your reasoning carefully. In all these equations, $y = (2\pi m\omega/h)^{1/2}$ and $\omega=(k/m)^{1/2}$. A is kind of normalization constant to be determined.

(i) $\Psi(x) = A(32y^5 - 160y^3 + 129y)\exp(y^2/2)$

(ii) $\Psi(x) = A(16y^4 - 48y^2 + 12)\exp(-y^2/2)$

(ii) $\Psi(x) = A(32y^5 + 160y^3 + 129y)\exp(-y^2/2)$

(iv) $\Psi(x) = A\exp(-y^2/2)$

(v) $\Psi(x) = A(2y)\exp(-y^2/2)$

(vi) $\Psi(x) = A(8y^3 + 4y^2 - 12y)\exp(-y^2/2)$

(vii) $\Psi(x) = A(32y^5 - 160y^3 + 129y)$

(b) (8 points)
What are the energy eigenvalues for this potential? What is the gound state energy?

(c) (10 points)
In all possible eigengunctions from part (a), choose one to calculate the normalization constant A, and the position expectation <x>. What is the energy of this eigenfunction? The following integrals may be useful:

$$\int_0^\infty e^{-t^2} t^{2z-1} dt = \frac{\Gamma(z)}{2} \text{ ; and } \Gamma(n) = (n-1)\Gamma(n-1)\text{; and } \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$