University of Kentucky Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics Fall 2003 **Final Examination**

Date: Dec 15, 2003 Time: 8:00-10:00 Answer all questions.

1. (Total: 40 points)

A particle of mass m is confined to a one dimensional region $0 \le x \le a$ with the potential

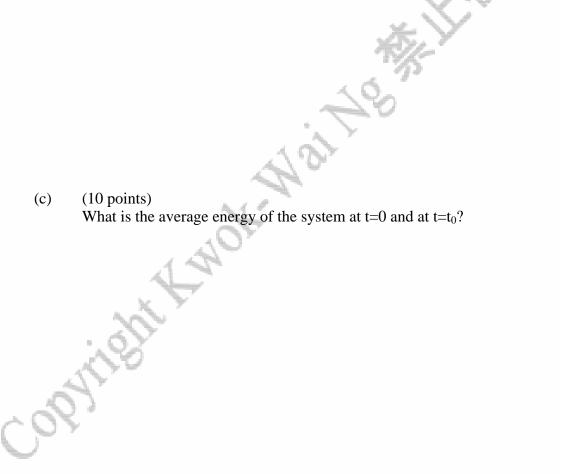
$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < a \\ \infty & x > 0 \end{cases}$$
 At t=0 its normalized wave function is

$$\Psi(x, t = 0) = \sqrt{\frac{8}{5a}} \left[1 + \cos\left(\frac{\pi x}{a}\right) \right] \sin\left(\frac{\pi x}{a}\right)$$

(6 points) (a)

Write down all the energy eigenstates $\psi_n(x)$ and the corresponding energy

(b) (12 points) What is the wave function at a later time $t = t_0$?



(d) (12 points) What is the probability that the particle is found in the left half of the box (i.e., in the region $0 \le x \le a/2$) at $t=t_0$?



2. (Total: 60 points)

For all parts of this problem, consider the case l = 1.

(a) (7 points)

With z-axis as the rotational axis, fill in the following commutation relation:

$[L_x, L_y] = $	
$[L_z, L_y] =$	
$[L^2, L_y] =$	
$[L^2, L_z] =$	
$[L_{+}, L_{x}] =$	
$[L_{+}, L_{z}] =$	
$[L_{+}, L_{-}] =$	

(b) (5 points)

With l = 1, how many eigenstates are there for L_z and what are their eigenvalues?

(c) (8 points)

With respect to the eigenstates of L_z , write down the operators L^2 and L_z in matrix form.

(d) (8 points) Write $L.L_+$ in terms of L^2 and L_z for the case when $m \ne 1$. What should be the value of $L.L_+$ if m=1?

 $\begin{array}{ll} \text{(e)} & \text{(8 points)} \\ \text{Construct operators L_{+} and L- in matrix form.} \end{array}$

(f) (8 points) Construct operators L_x and L_y in matrix form from your results in part (d).



(h) (8 points)
Use the operators in matrix form to prove any one of the commutation relations in part (a).

