University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2003
Final Examination

Date: Dec 15, 2003
Time: 8:00-10:00
Answer all questions.

1. (Total: 40 points)
A particle of mass m is confined to a one dimensional region $0 \le x \le a$ with the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < a \\ \infty & x > 0 \end{cases}$$

At t=0 its normalized wave function is

$$\Psi(x, t = 0) = \sqrt{\frac{8}{5a}}\left[1 + \cos\left(\frac{\pi x}{a}\right)\right]\sin\left(\frac{\pi x}{a}\right)$$

(a) (6 points)
Write down all the energy eigenstates $\psi_n(x)$ and the corresponding energy E_n.

(b) (12 points)
What is the wave function at a later time $t = t_0$?

(c) (10 points)
What is the average energy of the system at $t=0$ and at $t=t_0$?

(d) (12 points)

What is the probability that the particle is found in the left half of the box (i.e., in the region $0 \leq x \leq a/2$) at $t=t_0$?

2. (Total: 60 points)

For all parts of this problem, consider the case l =1.

(a) (7 points)

With z-axis as the rotational axis, fill in the following commutation relation:

$[L_x, L_y]$ = ______________________

$[L_z, L_y]$ = ______________________

$[L^2, L_y]$ = ______________________

$[L^2, L_z]$ = ______________________

$[L_+, L_x]$ = ______________________

$[L_+, L_z]$ = ______________________

$[L_+, L_-]$ = ______________________

(b) (5 points)

With l = 1, how many eigenstates are there for L_z and what are their eigenvalues?

(c) (8 points)

With respect to the eigenstates of L_z, write down the operators L^2 and L_z in matrix form.

(d) (8 points)

Write L_-L_+ in terms of L^2 and L_z for the case when $m \neq 1$. What should be the value of L_-L_+ if m=1?

(e) (8 points)

Construct operators L_+ and L- in matrix form.

(f) (8 points)
Construct operators L_x and L_y in matrix form from your results in part (d).

(g) (8 points)
What are the eigenvalues and eigenvectors of L_x?

(h) (8 points)
Use the operators in matrix form to prove any one of the commutation relations in part (a).