University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2003
Test 1

Answer all questions. Write down all work in detail.
Time allowed: 50 minutes

A particle of mass m is confined to a one dimensional region $-a/2 \le x \le a/2$ with an infinite potential

$$V(x) = \begin{cases} \infty & x \le -\frac{a}{2} \\ 0 & -\frac{a}{2} < x < \frac{a}{2} \\ \infty & x \ge \frac{a}{2} \end{cases}$$

The wave function at t=0 is given as:

$$\Psi(x, t=0) = A\,\psi_1(x) + B\,\psi_2(x)$$

where $\psi_1(x)$ is the ground state and $\psi_2(x)$ is the first excited state.

Answer all of the following.

(a) (8 points)
Write down all the energy eigenstates $\psi_n(x)$ and the corresponding energy E_n.

(b) (8 points)
If the probability of finding the particle in the ground state is 2 times the probability of finding it in the first excited state, determine the value of A and B by assuming they are real numbers. Use these values for the remaining of this problem.

(c) (10 points)
What is the probability that the particle is found in the right half of the box (i.e., in the region $0 \leq x \leq a/2$)?

(d) (10 points)
What is the probability of finding the particle in ground state ψ_1 at time t? How about the first excited state ψ_2? Do they depend on time?

(e) (8 points)
What is the probability of finding the particle in the left half of the box (i.e., in the region $0 \le x \le a/2$) at time t? Hint: Make use of your work in part (c), no need to do all integrations again!

(f) (8 points)
Is there any operator X that commute with the Hamiltonian of this problem? If so, calculate the expectation <X> of the state function $\Psi(x)$. Is <X> a constant over time? Briefly explain your answer.