University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2003
Test 2

Answer all questions. Write down all work in detail.
Time allowed: 50 minutes

Consider a double delta function potential

$$V(x) = -\frac{\hbar^2\lambda}{2ma}[\delta(x-a) + \delta(x+a)]$$

and $E < 0$.

Answer all of the following.

(a) (10 points)
Substitute the potential into the Schroedinger equation and simplify it.
Also write down the boundary conditions at $x = a$ and $x = -a$.

(b) (10 points)
The solution to the Schroedinger equation has to be either even or odd.
Explain why.

(c) (10 points)
Solve the Schroedinger equation up to a normalization and show that

$$\tanh \kappa a = \frac{\lambda}{\kappa a} - 1 \qquad \text{for even solution}$$

$$\coth \kappa a = \frac{\lambda}{\kappa a} - 1 \qquad \text{for odd solution}$$

where $\kappa = \sqrt{\frac{2m|E|}{\hbar^2}}$

(d) (10 points)
How many bound states are there for (i) small λ (ii) large λ?

Hint: You may want to re-write the eigenvalue condition given in part (c) as:

$$\tanh \kappa a = \frac{\lambda}{\kappa a} - 1 \qquad \text{for even solution}$$

$$\tanh \kappa a = \left(\frac{\lambda}{\kappa a} - 1 \right)^{-1} \qquad \text{for odd solution}$$