Name:________________________________

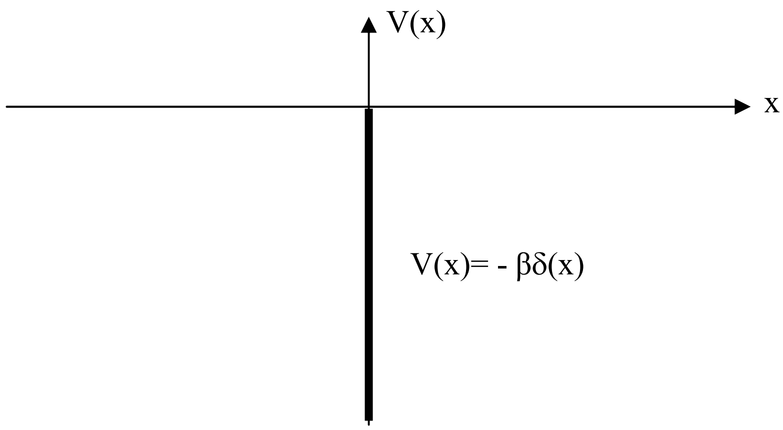
University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2004
Final Examination

Answer all four questions (total 200 points). Write down all work in detail.
Time allowed: 120 minutes
Merry Christmas and Happy New Year!

1. (50 points)

Consider a particle of mass moving under the influence of a delta function potential $V(x)= -\beta\delta(x)$ with energy $E<0$.



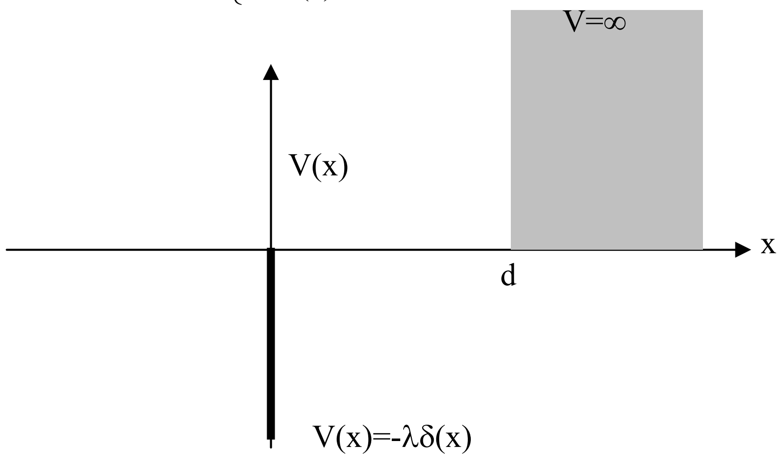
(a) (10 points)
Write down the boundary conditions for the eigenfunction at $x=0$.

(b) (20 points)
Solve for the eigenfunction(s) and the corresponding energy eigenvalue(s).
How many bound states are there?

(c) (20 points)

A wall is now placed at x=d so that the potential is given by

$$V(x) = \begin{cases} \infty & x > d \\ -\lambda\delta(c) & x \le d \end{cases}$$



Solve for the eigenfunction(s) and the corresponding energy eigenvalue(s) for the bound state(s). (To save time, you do not need to normalize the state function). Can you reduce your result to that of part (b) by letting d →∞ ?

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2. (50 points)

Consider a particle of mass m in a simple harmonic potential

$$V(x) = \frac{1}{2}kx^2$$

(a) (5 points)

Write down the ground state energy *in terms of m and k.*

(b) (5 points)

Which of the following functions is the ground state wave function of simple harmonic oscillation? Briefly explain (qualitatively) why you choose your answer.

A. $\frac{1}{\sqrt{2}}\sin\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\xi + \frac{1}{\sqrt{2}}\cos\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\xi$

B. $2\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\xi$

C. $2\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\xi\, e^{-\xi^2/2}$

D. $2\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\xi\, e^{-\xi/2}$

E. $\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\xi^2/2}$

F. $\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\xi/2}$

G. $\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} P_\ell^m(\xi)$

H. $\frac{1}{\sqrt{8}}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}(4\xi^2 - 2)e^{-\xi^2/2}$

In above, $\xi = \sqrt{\frac{m\omega}{\hbar}}\,x$, and P_ℓ^m is the Associated Legendre Polynomials.

(c) (10 points)
Calculate <x> and <p> for the ground state, where p is momentum.

(d) (15 points)
Calculate <x^2> and <p^2>. You may find the following integration useful:

$$\int_{-\infty}^{\infty} x^{2n} e^{-\alpha^2 x^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n \alpha^{2n+1}} \sqrt{\pi}$$

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(e) (15 points)

Calculate $\sigma_x\sigma_p$ to show that Uncertainty Principle is followed. Also show that <T> = <V>, where T is kinetic energy.

3. (50 points)

An electron is at rest in an oscillating magnetic field

$$\vec{B} = B_0 \cos(\omega t)\, \hat{k} \qquad (\hat{k} \text{ is the unit vector along the z - axis})$$

Where B_0 and ω are constants.

(a) (10points)
Construct the Hamiltonian matrix for this system.

(b) (20 points)
The electron starts out (at t=0) in the spin-up state wotj res[ect tp the x-axis (that is: $\chi(0) = \chi_+^{(x)}$). Determine $\chi(t)$ at any subsequent time from the time dependent Schroedinger equation.

(c) (15 points)
Find the probability of getting $+\hbar/2$, if you measure S_x.

(d) (5 points)
What is the minimum field (B_0) required to force a complete flip in S_x?

4. (50 points)

In doing this problem, you may find the following Clebsch-Gordon coefficient table for adding $|j_1,m_1>$ and $|j_2=1/2,m_2>$ into $|j, m_J>$:

	$m_1, m_2 = \frac{1}{2}$	$m_1, m_2 = -\frac{1}{2}$
$j = j_1 + \frac{1}{2}, m_J = m_1 + m_2$	$\sqrt{\frac{j_1 + m + \frac{1}{2}}{2j_1 + 1}}$	$\sqrt{\frac{j_1 - m + \frac{1}{2}}{2j_1 + 1}}$
$j = j_1 - \frac{1}{2}, m_J = m_1 + m_2$	$-\sqrt{\frac{j_1 - m + \frac{1}{2}}{2j_1 + 1}}$	$\sqrt{\frac{j_1 + m + \frac{1}{2}}{2j_1 + 1}}$

The state function of an electron in a hydrogen atom is described as

$$\Psi = \sqrt{\frac{2}{5}}\psi_{510} + \sqrt{\frac{3}{5}}\psi_{511}$$

and it is known that the electron is in a spin up state (i.e. m=1/2).

(a) (10 points)
What are the possible values of total angular momentum quantum number j and its z-component m_J?

(b) (10 points)
Make use of the above given Clebsch-Gordon coefficients, construct *two* tables for the cases (i) $m_1=0$, $m_2=1/2$, and (ii) $m_1=1$, $m_2=1/2$.

(c) (10 points)
Make use of your results in part (b), write Ψ in terms of the total angular momentum eigenvectors $|j, m_J\rangle$.

(d) (10 points)
If the total angular momentum is measured, what are the probabilities of obtaining (i) $j=1/2$ and (ii) $j=3/2$?

(e) (10 points)
If the z-component of the total angular momentum is measured, what are the probabilities of obtaining (i) $m_J=-3/2$, (ii) $m_J=-1/2$, (iii) $m_J=1/2$, and $m_J=3/2$?