University of Kentucky
Department of Physics and Astronomy

PHY 520 Introduction to Quantum Mechanics
Fall 2004
Test 1

Name: _____

Answer all questions. Write down all work in detail.
Time allowed: 50 minutes

1. (50 points)

A particle of mass m is confined to a one dimensional region $-a \le x \le a$ with an infinite potential

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 \le x \le a \\ \infty & x > a \end{cases}$$

The wave function at t=0 is given as:

$$\Psi(x, t=0) = A\left[\psi_1(x) + 2\ \psi_2(x)\right]$$

where $\psi_1(x)$ is the ground state and $\psi_2(x)$ is the first excited state.

Answer all of the following.

(a) (10 points)
Write down all the normalized energy eigenstates $\psi_n(x)$ and the corresponding energy E_n.

(b) (15 points)
Determine the value of A so that the state function $\Psi(x,0)$ is normalized.

(c) (15 points)
What is <E> at t=0?

(d) (10 points)
What is <E> at any other time t?

2. (50 points)

The first three energy eigenstates of a simple harmonic potential $V(x)=m\omega^2x^2/2$ are given as:

$$\psi_0 = \left(\frac{m\omega}{\hbar\pi}\right)^{\frac{1}{4}}(1)e^{-\xi^2/2}$$

$$\psi_1 = \sqrt{2}\left(\frac{m\omega}{\hbar\pi}\right)^{\frac{1}{4}}\xi e^{-\xi^2/2}$$

$$\psi_2 = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\hbar\pi}\right)^{\frac{1}{4}}(2\xi^2-1)e^{-\xi^2/2}$$

where

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\,x$$

You may find the following intergration useful for this problem :

$$\int_0^\infty e^{-\alpha u^2}du = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty u e^{-\alpha u^2}du = \frac{1}{2\alpha}$$

$$\int_0^\infty u^n e^{-\alpha u^2}du = \frac{n-1}{2\alpha}\int_0^\infty u^{n-2}e^{-\alpha u^2}du$$

The wavefunction os a particle at t=0 is given as:

$$\Psi(x,0) = \frac{1}{3}\left(\frac{m\omega}{\hbar\pi}\right)^{\frac{1}{4}}(4\xi^2-1)e^{-\xi^2/2}$$

(a) (20 points)

Write $\Psi(x,0)$ as a linear combination of ψ_0, ψ_1, and ψ_2.

(b) (15 points)
What is the wavefunction at time t (i.e. $\Psi(x,t)$) ?

(c) (15 points)
Calculate <E> at t=0 and at any other time t.