2. a. With respect to an orthogonal basis (vectors are perpendicular to each other and each vector has unit length), calculate the length of the vector

$$\begin{pmatrix} 1 \\ i \\ 1+i \end{pmatrix}$$

Solution:

$$(\text{Length})^2 = \begin{pmatrix} 1 \\ i \\ 1+i \end{pmatrix}^{+} \begin{pmatrix} 1 \\ i \\ 1+i \end{pmatrix}$$

$$= \begin{pmatrix} 1 & i & 1+i \end{pmatrix}^{*} \begin{pmatrix} 1 \\ i \\ 1+i \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -i & 1-i \end{pmatrix} \begin{pmatrix} 1 \\ i \\ 1+i \end{pmatrix}$$

$$= 1 \times 1 + (-i)(i) + (1-i)(1+i)$$

$$= 1 + 1 + 2 = 4$$

$$\therefore \text{Length} = \sqrt{4} = \underline{\underline{2}}$$

b. Write down the transpose of this matrix.
Solution:

$$\begin{pmatrix} 1 \\ i \\ 1+i \end{pmatrix}^{T} = \begin{pmatrix} 1 & i & 1+i \end{pmatrix}$$

Note the difference between transpose and adjoint.