Gasiorowicz 3rd edition April 24, 2008

P10-1.

Find the normalized eigenspinors for S_y.

Solution:

In S_z – representation :

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

The normalized eigenvectors of S_z is the same as that of matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. So we just need to work on the matrix without worrying the constant $\hbar$.

Let λ be the eigenvalues.

$$\det\begin{pmatrix} -\lambda & -i \\ i & -\lambda \end{pmatrix} = 0$$

$$\Rightarrow (\lambda^2 - 1) = 0$$

$$\Rightarrow \lambda = \pm 1$$

Let the eigenvectors be $\begin{pmatrix} u \\ v \end{pmatrix}$

Case1 : $\lambda = +1$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow \begin{cases} -iv = u \\ iu = v \end{cases} \quad \text{choose } u = 1, \text{ then } v = i$$

The eigenvector of this case is $\begin{pmatrix} 1 \\ i \end{pmatrix}$. It has a magnitude of $\sqrt{|1|^2 + |i|^2} = \sqrt{2}$. Normalizing it, the eigenvector becomes $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

Case1 : $\lambda = -1$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = -\begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow \begin{cases} -iv = -u \\ iu = -v \end{cases} \quad \text{choose } u = 1, \text{ then } v = -i$$

The eigenvector of this case is $\begin{pmatrix} 1 \\ -i \end{pmatrix}$. It has a magnitude of $\sqrt{|1|^2 + |-i|^2} = \sqrt{2}$. Normalizing it, the eigenvector becomes $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Hence, the two eigenvectors of S_y in S_z – representation are $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$.