Gasiorowicz 3rd edition Jan 25, 2008

P2-15.

Consider the wave function obtained in the first example, 2N(sin kx)/x For what value of N will it be normalized? [Hint: A useful integral is $\int_{-\infty}^{\infty}\left(\frac{\sin t}{t}\right)^2 = \pi$.]

Solution:

$$\psi(x) = 2N\frac{\sin kx}{x}$$

$$\int_{-\infty}^{\infty}\psi^*(x)\psi(x)dx = 1 \Rightarrow \int_{-\infty}^{\infty}\left(2N\frac{\sin kx}{x}\right)^2 dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} 4N^2\left(\frac{\sin kx}{x}\right)^2 dx = 1$$

$$\Rightarrow 4N^2\int_{-\infty}^{\infty}\left(\frac{\sin kx}{x}\right)^2 dx = 1$$

$$\Rightarrow 4N^2\pi = 1$$

$$\Rightarrow N^2 = \frac{1}{4\pi}$$

$$\Rightarrow N = \underline{\underline{\frac{1}{2\sqrt{\pi}}}}$$