Gasiorowicz 3rd edition Feb 6, 2008

P2-18.

Use the definition $(\Delta x)^2 = <x^2> - <x>^2$ and $(\Delta p)^2 = <p^2> - <p>^2$ with the results of Problem 16 and 17 to show that $\Delta p \Delta x > \hbar/2$.

Solution:

Problem 16:

$$<x> = 0$$

$$<x^2> = \frac{1}{2\alpha}$$

$$\therefore (\Delta x)^2 = \frac{1}{2\alpha} - 0 = \frac{1}{2\alpha}$$

$$\Delta x = \frac{1}{\sqrt{2\alpha}}$$

$$\Delta p = \sqrt{\frac{\alpha}{2}}\hbar$$

Problem 17:

$$<p> = 0$$

$$<p^2> = \frac{(\alpha/\pi)^{1/2}}{\alpha\hbar} \cdot \frac{\alpha\hbar^2}{2}\sqrt{\alpha\hbar^2\pi} = \frac{\alpha\hbar^2}{2}$$

$$\therefore (\Delta p)^2 = \frac{\alpha\hbar^2}{2} - 0 = \frac{\alpha\hbar^2}{2}$$

$$\Delta p = \sqrt{\frac{\alpha}{2}}\hbar$$

$$\Delta p \Delta x = \sqrt{\frac{\alpha}{2}}\hbar \cdot \frac{1}{2\alpha} = \frac{\hbar}{2}$$