

The wave function for a particle is given by

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

What flux does this represent?

Solution:

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$\begin{aligned} \therefore j &= \frac{\hbar}{2mi} [\psi^* \nabla \psi - \psi \nabla \psi^*] \\ &= \frac{\hbar}{2mi} \left[(Ae^{ikx} + Be^{-ikx})^* \frac{d}{dx} (Ae^{ikx} + Be^{-ikx}) - (Ae^{ikx} + Be^{-ikx}) \frac{d}{dx} (Ae^{ikx} + Be^{-ikx})^* \right] \\ &= \frac{\hbar}{2mi} \left[(A^* e^{-ikx} + B^* e^{ikx}) \frac{d}{dx} (Ae^{ikx} + Be^{-ikx}) - (Ae^{ikx} + Be^{-ikx}) \frac{d}{dx} (A^* e^{-ikx} + B^* e^{ikx}) \right] \\ &= \frac{\hbar}{2mi} \left[(A^* e^{-ikx} + B^* e^{ikx}) (ikAe^{ikx} - ikBe^{-ikx}) - (Ae^{ikx} + Be^{-ikx}) (-ikA^* e^{-ikx} + ikB^* e^{ikx}) \right] \\ &= \frac{\hbar}{2mi} \left[(ikA^2 - ikB^2 + ikAB^* e^{i2kx} - ikA^* Be^{-i2kx}) - (-ikA^2 + ikB^2 - ikA^* Be^{-i2kx} + ikAB^* e^{i2kx}) \right] \\ &= \frac{\hbar}{2mi} (2ikA^2 - 2ikB^2) \\ &= \frac{\hbar k}{m} (A^2 - B^2) \end{aligned}$$