Gasiorowicz 3rd edition Feb 26, 2008
P3-16.

What is the flux associated with a particle described by the wave function

$$\psi(x) = u(x)e^{ikx}$$

where u(x) os a real function?

Solution:

$$\psi(x) = u(x)e^{ikx}$$

$$\therefore j = \frac{\hbar}{2mi}\left[\psi^* \nabla\psi - \psi\nabla\psi^*\right]$$

$$= \frac{\hbar}{2mi}\left\{[u(x)e^{ikx}]^* \frac{d}{dx}[u(x)e^{ikx}] - [u(x)e^{ikx}]\frac{d}{dx}[u(x)e^{ikx}]^*\right\}$$

$$= \frac{\hbar}{2mi}\left\{[u(x)e^{-ikx}]\frac{d}{dx}[u(x)e^{ikx}] - [u(x)e^{ikx}]\frac{d}{dx}[u(x)e^{-ikx}]\right\}$$

$$= \frac{\hbar}{2mi}\left\{[u(x)e^{-ikx}][u'(x)e^{ikx} + iku(x)e^{ikx}] - [u(x)e^{ikx}][u'(x)e^{-ikx} - iku(x)e^{-ikx}]\right\}$$

$$= \frac{\hbar}{2mi}\left\{u(x)[u'(x) + iku(x)] - u(x)[u'(x) - iku(x)]\right\}$$

$$= \frac{\hbar}{2mi}\left[u(x)u'(x) + iku^2(x) - u(x)u'(x) + iku^2(x)\right] \qquad (u^2(x) = [u(x)]^2)$$

$$= \frac{\hbar}{2mi}\left[2iku^2(x)\right]$$

$$= \underline{\underline{\frac{\hbar k}{m}u^2(x)}}$$