Gasiorowicz 3rd edition   Feb 17, 2008

P3-9.

A particle is known to be localized in the left half of a box with sides at x=±a/2. woth wave function

$$\psi(x) = \begin{cases} = \sqrt{\dfrac{2}{a}} & \dfrac{-a}{2} < x < 0 \\ = 0 & 0 < x < \dfrac{a}{2} \end{cases}$$

(a) Will the particle remain localized at later times?

(b) Calculate the probability that an energy measurement yields the ground state energy; the energy of the first excited state.

Solution:

(a) Since $\psi(x)$ is not an energy eigenstate, so it will not remain "stationary" (up to a phase factor) over time. Hence the particle will not remain localized at later times.

$$\text{(b)}\ |\,\psi > = \begin{cases} = \sqrt{\dfrac{2}{a}} & \dfrac{-a}{2} < x < 0 \\ = 0 & 0 < x < \dfrac{a}{2} \end{cases}$$

$$\text{If}\ |\,\psi > = \sum_{i=1}^{\infty} a_n\ |\,u_n(x) >$$

$$a_n = < u_n(x)\,|\,\psi > = \int_{-a/2}^{a/2} u_n{}^*(x)\,\psi(x)dx$$

$$= \sqrt{\frac{2}{a}} \int_{-a/2}^{0} u_n{}^*(x)\,dx$$

$$= \begin{cases} \sqrt{\dfrac{2}{a}} \displaystyle\int_{-a/2}^{0} \sqrt{\dfrac{2}{a}} \cos\dfrac{n\pi x}{a}\,dx & \text{odd n} \\ \sqrt{\dfrac{2}{a}} \displaystyle\int_{-a/2}^{0} \sqrt{\dfrac{2}{a}} \sin\dfrac{n\pi x}{a}\,dx & \text{even n} \end{cases}$$

$$= \begin{cases} \left[\dfrac{2}{a}\dfrac{a}{n\pi}\sin\dfrac{n\pi x}{a}\right]_{-a/2}^{0} & \text{odd n} \\ \left[-\dfrac{2}{a}\dfrac{a}{n\pi}\cos\dfrac{n\pi x}{a}\right]_{-a/2}^{0} & \text{even n} \end{cases}$$

$$a_n = \begin{cases} (-1)^{\frac{n-1}{2}} \dfrac{2}{n\pi} & \text{odd } n \\ \left[-1+(-1)^{\frac{n}{2}}\right]\dfrac{2}{n\pi} & \text{even } n \end{cases}$$

$n = 1$ for ground state.

$$a_1 = (-1)^{\frac{1-1}{2}} \frac{2}{\pi} = \frac{2}{\pi}$$

$\therefore$ Probability for ground state energy to be measured $= {a_1}^2 = \dfrac{4}{\pi^2}$

$n = 2$ for first excited state.

$$a_2 = \left[-1+(-1)^{\frac{2}{2}}\right]\frac{a}{n\pi} = -2 \cdot \frac{2}{2\pi} = -\frac{2}{\pi}$$

$\therefore$ Probability for ground state energy to be measured $= {a_2}^2 = \left(-\dfrac{2}{\pi}\right)^2 = \dfrac{4}{\pi^2}$