Gasiorowicz 3rd edition April 1, 2008
P5-11.

Operators that do not commute obey a number of relationships. Prove the following:
(a) If A and B are hermitian, then i[A,B] us also Hermitian.
(b) [AB.C] = A[B,C]+[A,C]B
(c) [A,[B,C]]+ [B,[C,A]]+ [C,[A,B]]=0

Solution:

(a) A and B are Hermitian $\Rightarrow$ $A^+ = A$ and $B^+ = B$

$$i[A,B] = i(AB - BA)$$

$$\therefore (i[A,B])^+ = \{ i(AB - BA) \}^+$$
$$= -i\{(AB)^+ - (BA)^+\}$$
$$= -i\{(B^+A^+) - (B^+A^+)\}$$
$$= -i\{(BA) - (AB)\} \qquad (\because A^+ = A \text{ and } B^+ = B)$$
$$= i\{(AB) - (BA)\}$$
$$= i[A,B]$$

$\therefore i[A,B]$ is Hermitian.

(b)
$$[AB,C] = ABC - CAB$$
$$= ABC - ACB + ACB - CAB$$
$$= (ABC - ACB) + (ACB - CAB)$$
$$= A(BC - CB) + (AC - CA)B$$
$$= A[B,C] + [A,C]B$$

(c)
$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = [A, BC - CB] + [B, CA - AC] + [C, AB - BA]$$
$$= [A,BC] - [A,CB] + [B,CA] - [B,AC] + [C,AB] - [C,BA]$$
$$= (ABC - BCA) - (ACB - CBA) + (BCA - CAB) - (BAC - ACB)$$
$$+ (CAB - ABC) - (CBA - BAC)$$
$$= 0$$