Gasiorowicz 3rd edition April 1, 2008

P5-13.

Consider the Hermitian operator H that has the property

$$H^4=1$$

What are the eigenvalues of the operator H? What are the eigenvalues of H if it is not restricted to being Hermitian?

Solution:

Let $H|\psi>=E|\psi>$ where E is the eigenvalue.

$$\therefore H^2|\psi>=H(E|\psi>)=E(H|\psi>)=E^2|\psi>$$

$$H^3|\psi>=H(E^2|\psi>)=E^2(H|\psi>)=E^3|\psi>$$

$$H^4|\psi>=H(E^3|\psi>)=E^3(H|\psi>)=E^4|\psi>$$

$$H^4|\psi>=|\psi> \Rightarrow E^4=1$$

$E^4=1$ has four roots: $E=1, -1, i, -i$

Since H is Hermitian, E has to be real. $\therefore$ The eigenvalues of Hermitian H are 1 and -1.

If H is not restricted to Hermitian, H has four eigenvalues, 1, -1, i, -i.