Gasiorowicz 3rd edition April 14, 2008

P5-17.

An electron in an oscillating electric field is described by the Hamiltonian operator

$$H = \frac{p^2}{2m} - (eE_0 \cos\omega t)x$$

Calculate expressions for the time dependence of <x>, <p> and <H>.

Solution:

$$\frac{d}{dt} < p >= \left\langle \frac{\partial}{\partial t} p \right\rangle + \frac{i}{\hbar} < [H,p] >$$

$$= \frac{i}{\hbar} < [\frac{p^2}{2m} - (eE_0 \cos\omega t)x, p] >$$

$$= \frac{i}{\hbar} < [-(eE_0 \cos\omega t)x, p] >$$

$$= \frac{i}{\hbar} \cdot -(eE_0 \cos\omega t) \cdot < [x,p] >$$

$$= \frac{i}{\hbar} \cdot -(eE_0 \cos\omega t) \cdot i\hbar$$

$$= eE_0 \cos\omega t$$

Integrating,

$$< p >= \int eE_0 \cos\omega t \, dt = \underline{\underline{\frac{eE_0}{\omega} \sin\omega t + < p >_0}} \quad \text{where} < p >_0 \text{ is} < p > \text{at } t = 0$$

$$\frac{d}{dt} < x >= \left\langle \frac{\partial}{\partial t} x \right\rangle + \frac{i}{\hbar} < [H,x] >$$

$$= \frac{i}{\hbar} < [\frac{p^2}{2m} - (eE_0 \cos\omega t)x, x] >$$

$$= \frac{i}{\hbar} < [\frac{p^2}{2m}, x] >$$

$$= \frac{2i}{\hbar} < [\frac{p}{2m}, x] p >$$

$$= \frac{2i}{\hbar} \cdot \frac{1}{2m} \cdot -i\hbar < p >$$

$$= \frac{< p >}{m}$$

$$= \frac{1}{m}\left(\frac{eE_0}{\omega} \sin\omega t + < p >_0 \right)$$

$$< x >= \int \frac{1}{m}\left(\frac{eE_0}{\omega} \sin\omega t + < p >_0 \right) dt = \frac{1}{m}\left[\frac{eE_0}{\omega^2} \cos\omega t + < p >_0 t + \left(m < x >_0 - \frac{eE_0}{\omega^2} \right) \right]$$

$$= \underline{\underline{\left[\frac{eE_0}{m\omega^2}(\cos\omega t - 1) + \frac{< p >_0}{m} t + < x >_0 \right]}}$$

$$\text{where} < x >_0 \text{ is} < x > \text{at } t = 0$$

$$\frac{d}{dt} < H > = \left\langle \frac{\partial}{\partial t} H \right\rangle + \frac{i}{\hbar} < [H, H] >$$

$$= < (eE_0 \omega \sin\omega t)\, x > \; + \; 0$$

$$= (eE_0 \omega \sin\omega t) < x >$$

$$= \underline{\underline{eE_0 \omega \sin\omega t \cdot \left[\frac{eE_0}{m\omega^2}\left(\cos\omega t - 1\right) + \frac{< p >_0}{m} t + < x >_0 \right]}}$$