Gasiorowicz 3rd edition April 22, 2008

P7-10.

Consider a state of total angular momentum l=2. What are the eigenvalues of the operators (a)L_z, (b)$\frac{3}{5}L_x - \frac{4}{5}L_y$, and (c)$2L_x - 6L_y + 3L_z$?

Solution:

(a). The eigenvalues of L_z are $-2\hbar, -\hbar, 0, \hbar$, and $2\hbar$.

(b)

$$\text{Choose } \frac{\frac{3}{5}\hat{x} + \frac{4}{5}\hat{y}}{\sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}} = \frac{3}{5}\hat{x} + \frac{4}{5}\hat{y} \text{ as the new z-axis.}$$

$$\therefore \left(\frac{3}{5}L_x - \frac{4}{5}L_y\right) = \left(\frac{3}{5}\hat{x} + \frac{4}{5}\hat{y}\right)\cdot \vec{L} = L_z$$

$\therefore$ The eigenvalues of $\frac{3}{5}L_x - \frac{4}{5}L_y$ are $-2\hbar, -\hbar, 0, \hbar$, and $2\hbar$.

(c)

$$\text{Choose } \frac{-2\hat{x} - 6\hat{y} + 3\hat{z}}{\sqrt{(-2)^2 + (-6)^2 + (3)^2}} = -\frac{2}{7}\hat{x} - \frac{6}{7}\hat{y} + \frac{3}{7}\hat{z} \text{ as the new z-axis.}$$

$$\therefore 2L_x - 6L_y + 3L_z = 7\cdot\left(-\frac{2}{7}\hat{x} - \frac{6}{7}\hat{y} + \frac{3}{7}\hat{z}\right)\cdot \vec{L} = 2L_z$$

$\therefore$ The eigenvalues of $2L_x - 6L_y + 3L_z$ are $-14\hbar, -7\hbar, 0, 7\hbar$, and $14\hbar$.