Gasiorowicz 3rd edition April 14, 2008

P7-3.

Calculate $<\ell, m_1 | L_x | \ell, m_2>$ and $<\ell, m_1 | L_y | \ell, m_2>$.

Solution:

$$L_x = \frac{1}{2}(L_+ + L_-)$$

$$L_x | \ell, m_2 >= \frac{1}{2}(L_+ + L_-) | \ell, m_2 >$$

$$= \frac{1}{2}(L_+ | \ell, m_2 > + L_- | \ell, m_2 >)$$

$$= \frac{1}{2}(\sqrt{\ell(\ell+1) - m_2(m_2+1)}\, | \ell, m_2+1 > + \sqrt{\ell(\ell+1) - m_2(m_2-1)}\, | \ell, m_2-1 >)$$

$$\therefore < \ell, m_1 | L_x | \ell, m_2 >$$

$$= \frac{1}{2}(\sqrt{\ell(\ell+1) - m_2(m_2+1)} < \ell, m_1 | \ell, m_2+1 > + \sqrt{\ell(\ell+1) - m_2(m_2-1)} < \ell, m_1 | \ell, m_2-1 >)$$

$$= \frac{1}{2}(\sqrt{\ell(\ell+1) - m_2(m_2+1)}\, \delta_{m_1, m_2+1} < \ell, m_1 | \ell, m_2+1 > + \sqrt{\ell(\ell+1) - m_2(m_2-1)}\;\; \delta_{m_1, m_2-1})$$

$$L_y = \frac{i}{2}(L_- - L_+)$$

$$L_y | \ell, m_2 >= \frac{i}{2}(L_- - L_+) | \ell, m_2 >$$

$$= \frac{i}{2}(L_- | \ell, m_2 > - L_+ | \ell, m_2 >)$$

$$= \frac{i}{2}(\sqrt{\ell(\ell+1) - m_2(m_2-1)}\, | \ell, m_2-1 > - \sqrt{\ell(\ell+1) - m_2(m_2+1)}\, | \ell, m_2+1 >)$$

$$\therefore < \ell, m_1 | L_y | \ell, m_2 >$$

$$= \frac{i}{2}(\sqrt{\ell(\ell+1) - m_2(m_2-1)} < \ell, m_1 | \ell, m_2-1 > - \sqrt{\ell(\ell+1) - m_2(m_2+1)} < \ell, m_1 | \ell, m_2+1 >)$$

$$= \frac{i}{2}(\sqrt{\ell(\ell+1) - m_2(m_2-1)}\;\; \delta_{m_1, m_2-1} - \sqrt{\ell(\ell+1) - m_2(m_2+1)}\, \delta_{m_1, m_2+1})$$

.