

A system is described by the Hamiltonian

$$H = \frac{L^2}{2I} + \alpha L_z$$

What is the energy spectrum of the system?

Solution:

$$\begin{aligned} H | \ell, m \rangle &= \frac{L^2}{2I} | \ell, m \rangle + \alpha L_z | \ell, m \rangle \\ &= \frac{\ell(\ell+1)\hbar^2}{2I} | \ell, m \rangle + \alpha m \hbar | \ell, m \rangle \\ &= \left(\frac{\ell(\ell+1)\hbar^2}{2I} + \alpha m \hbar \right) | \ell, m \rangle \end{aligned}$$

Hence the energy spectrum is

$$E = \frac{\ell(\ell+1)\hbar^2}{2I} + \alpha m \hbar$$