Gasiorowicz 3rd edition April 25, 2008
P8-6

Compare the wavelengths of the 2P → 1S transition in (1 hydrogen, (2) deuterium (nuclear mass = 2× proton mass), (3) positronium (a bund state of an electron and a positron, whose mass is the same as that of an electron).

Solution:

The energy level of a hydrogen atom can be calculated as

$$E_n = -\frac{1}{2}\mu c^2 \frac{\left(Z\alpha^2\right)}{n^2}$$

For $2 \to 1$ transition,

$$\Delta E = \frac{1}{2}\mu c^2\left(Z\alpha^2\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \left[\frac{3}{8}c^2\left(Z\alpha^2\right)\right]\mu$$

With $Z = 1$ and $\Delta E = \frac{hc}{\lambda}$,

$$\frac{hc}{\lambda} = \left[\frac{3}{8}c^2\alpha^2\right]\mu \Rightarrow \lambda = \frac{8h}{3c\alpha^2\mu} = \frac{8\times 6.626\times 10^{-34}}{3\times 2.998\times 10^8 \times \left(\frac{1}{137}\right)^2}\cdot\frac{1}{\mu}$$

$$= \frac{1.1062\times 10^{-37}}{\mu}$$

$$\text{(a)}\ \mu = \frac{m_e m_p}{m_e + m_p} = \frac{9.1095\times 10^{-31}\times 1.6726\times 10^{-27}}{9.1095\times 10^{-31} + 1.6726\times 10^{-27}} = 9.1045\times 10^{-31}$$

$$\therefore \lambda = \frac{1.1062\times 10^{-37}}{9.1045\times 10^{-31}} = \underline{\underline{1.215\times 10^{-7}\ \text{m or } 0.1215\mu\text{m}}}$$

$$\text{(b)}\mu = \frac{2m_e m_p}{m_e + 2m_p} = \frac{2\times 9.1095\times 10^{-31}\times 1.6726\times 10^{-27}}{9.1095\times 10^{-31} + 2\times 1.6726\times 10^{-27}} = 9.1070\times 10^{-31}$$

$$\therefore \lambda = \frac{1.1062\times 10^{-37}}{9.1070\times 10^{-31}} = \underline{\underline{1.215\times 10^{-7}\ \text{m or } 0.1215\mu\text{m}}}$$

There is no significant difference between (a) and (b) because proton is too heavy in comparison with electron mass, and the reduce mass is nearly the same for both cases.

$$\text{(b)}\mu = \frac{m_e m_e}{m_e + m_e} = \frac{1}{2}m_e = \frac{9.1095\times 10^{-31}}{2} = 4.5548\times 10^{-31}$$

$$\therefore \lambda = \frac{1.1062\times 10^{-37}}{4.5548\times 10^{-31}} = \underline{\underline{2.429\times 10^{-7}\ \text{m or } 0.2429\mu\text{m}}}$$