

Ex. 2-14.

(a) $\langle \phi | \hat{A} | \psi \rangle \langle \psi |$ is a bra.

$(\langle \phi | \hat{A} | \psi \rangle \langle \psi |)^+ = |\psi \rangle \langle \psi | \hat{A}^+ | \phi \rangle$ is a ket.

(b) $\hat{A} | \psi \rangle \langle \phi |$ is an operator.

$(\hat{A} | \psi \rangle \langle \phi |)^+ = |\phi \rangle \langle \psi | \hat{A}^+$ which is an operator again.

(c) $\langle \phi | \hat{A} | \psi \rangle \neq |\psi \rangle \langle \phi | \hat{A}$ is an operator.

$$\begin{aligned} & (\langle \phi | \hat{A} | \psi \rangle | \psi \rangle \langle \phi | \hat{A})^+ \\ &= \hat{A}^+ | \phi \rangle \langle \psi | \underbrace{\langle \psi | \hat{A}^+ | \phi \rangle}_{\text{number}} \end{aligned}$$

$= \langle \psi | \hat{A}^+ | \phi \rangle \hat{A}^+ | \phi \rangle \langle \psi |$ which is an operator again.

(d) $\langle \psi | \hat{A} | \phi \rangle | \phi \rangle + i \hat{A} | \psi \rangle$ is a ket.

$$\begin{aligned} & (\langle \psi | \hat{A} | \phi \rangle | \phi \rangle + i \hat{A} | \psi \rangle)^+ \\ &= \langle \phi | \langle \phi | \hat{A}^+ | \psi \rangle - i \langle \psi | \hat{A}^+ | \phi \rangle, \text{ a bra.} \end{aligned}$$

(e) $| \phi \rangle \langle \phi | \hat{A} - i \hat{A} | \psi \rangle \langle \psi |$ is an operator.

$$\begin{aligned} & (| \phi \rangle \langle \phi | \hat{A} - i \hat{A} | \psi \rangle \langle \psi |)^+ \\ &= \hat{A}^+ | \phi \rangle \langle \phi | + i | \psi \rangle \langle \psi | \hat{A}^+, \text{ which is an operator.} \end{aligned}$$